Parallel and Perpendicular Lines

Parallel Lines:

If two nonvertical lines have the same slope, then they are parallel.

All vertical lines are parallel.

Example 1:

Find the slope of a line parallel to the given line.

$y = \frac{1}{4}x + 2$

We know that parallel lines have the same slope. The slope of the line given to us is $\frac{1}{4}$.
Therefore, the slope of a line parallel to the given line would be $\frac{1}{4}$.

Slope: $\frac{1}{4}$

Example 2:

Find the slope of a line parallel to the given line.

$x + 2y = 6$

In order to find the slope of a line parallel, we need to know the slope of this line. So, let's solve for y.

$x + 2y = 6$

$-x \qquad -x$

$2y = -x + 6$

$\frac{2y}{2} = -\frac{x}{2} + \frac{6}{2}$

$y = -\frac{1}{2}x + 3$

We know that parallel lines have the same slope. The slope of the line given to us is $-\frac{1}{2}$.
Therefore, the slope of a line parallel to the given line would be $-\frac{1}{2}$.

Slope: $-\frac{1}{2}$

Perpendicular Lines:

If the product of the slopes of two nonvertical lines is -1, then the lines are perpendicular.

If the slopes are negative reciprocals of each other than the lines are perpendicular.

Vertical and horizontal lines are perpendicular.

Negative Reciprocal:

To be reciprocal just means that numerators and denominators of the fraction represented by the slope are switched. To be negative means that the signs are opposite.

Find the negative reciprocals of the following numbers:

$$-2,\ \frac{1}{4},\ -\frac{3}{2},\ 1,\ \frac{5}{4},\ -\frac{1}{2}$$

$-2 = -\frac{2}{1}$ To find the negative reciprocal, we flip the fraction and change the sign: $\frac{1}{2}$

$\frac{1}{4}$ To find the negative reciprocal, we flip the fraction and change the sign: $-\frac{4}{1} = -4$

$-\frac{3}{2}$ To find the negative reciprocal, we flip the fraction and change the sign: $\frac{2}{3}$

$1 = \frac{1}{1}$ To find the negative reciprocal, we flip the fraction and change the sign: $-\frac{1}{1} = -1$

$\frac{5}{4}$ To find the negative reciprocal, we flip the fraction and change the sign: $-\frac{4}{5}$

$-\frac{1}{2}$ To find the negative reciprocal, we flip the fraction and change the sign: $\frac{2}{1} = 2$

Example 3:

Find the slope of a line perpendicular to the given line.

$y = 5x - 1$

We know that perpendicular lines have negative reciprocal slopes. The slope of the line given to us is 5. Therefore, the slope of a line parallel to the given line would be $-\frac{1}{5}$.

Slope: $-\frac{1}{5}$

Example 4:

Find the slope of a line perpendicular to the given line.

$x - y = 1$

In order to find the slope of a line perpendicular, we need to know the slope of this line. So, let's solve for y.

$x - y = 1$

$-x \quad -x$

$-y = -x + 1$

$\frac{-y}{-1} = -\frac{x}{-1} + \frac{1}{-1}$

$y = x - 1$

We know that perpendicular lines have negative reciprocal slopes. The slope of the line given to us is 1. Therefore, the slope of a line parallel to the given line would be -1.

Slope: -1

Example 5:

Determine whether the lines formed by the points are parallel, perpendicular, or neither.

$A(-1, -2)$, $B(3, 2)$, $C(-4, -3)$, $D(1, 2)$

AB and CD

A B

x	y
-1	-2
3	2

$+4$ $+4$

$\frac{\Delta y}{\Delta x} = \frac{4}{4} = 1$

CD

x	y
-4	-3
1	2

$+5$ $+5$

$\frac{\Delta y}{\Delta x} = \frac{5}{5} = 1$

Since the slopes are the same, the lines are **parallel**.

Example 6:

Determine whether the lines formed by the points are parallel, perpendicular, or neither.

$A(-4,\ 4),\ B(1,\ -3),\ C(-5,\ 5),\ D(2,\ 0)$

AB and CD

A B

x	y
-4	4
1	-3

$+5$ $\quad -7$

$\frac{\Delta y}{\Delta x} = -\frac{7}{5}$

CD

x	y
-5	5
2	0

$+7$ $\quad -5$

$\frac{\Delta y}{\Delta x} = -\frac{5}{7}$

Since the slopes are not the same, the lines are not parallel.

The slopes are reciprocals, but not negative reciprocals, so the lines are not perpendicular.

Neither.

Example 7:

Determine whether the lines formed by the points are parallel, perpendicular, or neither.

$A(1,\ 4),\ B(3,\ -3),\ C(-3,\ -1),\ D(4,\ 1)$

AB and CD

A B

x	y
1	4
3	-3

$+2$ $\quad -7$

$\frac{\Delta y}{\Delta x} = -\frac{7}{2}$

CD

x	y
-3	-1
4	1

$+7$ $\quad +2$

$\frac{\Delta y}{\Delta x} = \frac{2}{7}$

Since the slopes are not the same, the lines are not parallel.

The slopes are negative reciprocals, so the lines are **perpendicular**.

Example 8:

Write the slope-intercept form of an equation for the line that passes through $(-1, -2)$ and is parallel to the graph of $y = -3x - 2$.

We know that the slope of a line parallel to this line would be the same as the slope of this line. So, since the slope of this line is -3, so is the slope of our line.

**Note: The y-intercept of the line given to use tells us where that line crosses the y-axis. We don't want our line to cross the y-axis at the same place as this line. If it did, it would be right on top of this line. We want our line to never intersect this line.

$y = mx + b$

$y = -3x + b$ *I'm only replacing slope because I need to find where the new line would cross the y-axis.

We use the point $(-1, -2)$ to replace the x- and y-values.

$-2 = -3(-1) + b$

$-2 = 3 + b$

$-3 \quad -3$

$-5 = b$

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$\boldsymbol{y = -3x - 5}$

Example 9:

Write the slope-intercept form of an equation for the line that passes through $(4, 0)$ and is parallel to the graph of $4x - 3y = -15$.

In order to find the slope of a line parallel, we need to know the slope of this line. So, let's solve for y.

$4x - 3y = -15$

$-4x \qquad -4x$

$-3y = -4x - 15$

$$\frac{-3y}{-3} - \frac{-4x}{-3} - \frac{15}{-3}$$

$$y = \frac{4}{3}x + 5$$

We know that the slope of a line parallel to this line would be the same as the slope of this line. So, since the slope of this line is $\frac{4}{3}$, so is the slope of our line.

**Note: The y-intercept of the line given to use tells us where that line crosses the y-axis. We don't want our line to cross the y-axis at the same place as this line. If it did, it would be right on top of this line. We want our line to never intersect this line.

$y = mx + b$

$y = \frac{4}{3}x + b$ *I'm only replacing slope because I need to find where the new line would cross the y-axis.

We use the point $(4, 0)$ to replace the x- and y-values.

$0 = -3(4) + b$

$0 = -12 + b$

$+12 + 12$

$12 = b$

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$$\boldsymbol{y = \frac{4}{3}x + 12}$$

Example 10:

Write the slope-intercept form of an equation for the line that passes through $(1,\ 1)$ and is perpendicular to the graph of $y = -\frac{1}{3}x + 1$.

We know that perpendicular lines have negative reciprocal slopes. The slope of the line given to us is $-\frac{1}{3}$. Therefore, the slope of a line parallel to the given line would be 3.

**Note: The y-intercept of the line given to use tells us where that line crosses the y-axis. We don't necessarily want our line to cross the y-axis at the same place as this line. It could, but we don't want to make that assumption.

$y = mx + b$

$y = 3x + b$ *I'm only replacing slope because I need to find where the new line would cross the y-axis.

We use the point $(1, 1)$ to replace the x- and y-values.

$1 = 3(1) + b$

$1 = 3 + b$

$-3 - 3$

$-2 = b$

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$\boldsymbol{y = 3x - 2}$

Example 11:

Write the slope-intercept form of an equation for the line that passes through $(-3, -2)$ and is perpendicular to the graph of $x + 4y = 12$.

In order to find the slope of a line perpendicular, we need to know the slope of this line. So, let's solve for y.

$x + 4y = 12$

$-x \quad -x$

$4y = -x + 12$

$\frac{4y}{4} = \frac{-x}{4} + \frac{12}{4}$

$y = -\frac{1}{4}x + 3$

We know that perpendicular lines have negative reciprocal slopes. The slope of the line given to us is $-\frac{1}{4}$. Therefore, the slope of a line parallel to the given line would be 4.

**Note: The y-intercept of the line given to use tells us where that line crosses the y-axis. We don't necessarily want our line to cross the y-axis at the same place as this line. It could, but we don't want to make that assumption.

$y = mx + b$

$y = 4x + b$ *I'm only replacing slope because I need to find where the new line would cross the y-axis.

We use the point $(-3, -2)$ to replace the x- and y-values.

$-2 = 4(-3) + b$

$-2 = -12 + b$

$+12 \ \ +12$

$10 = b$

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$\boldsymbol{y = 4x + 10}$

Example 12:

Write the slope-intercept form of an equation for the line that passes through $(-4, \ -5)$ and is parallel to the graph of $x = 0$.

$x = 0$ is the graph of a vertical line. We know that a parallel line to a vertical line would be another vertical line. So, our graph will also be the equation of a vertical line which is $x = \#$. Since we know that the line passes through $(-4, \ -5)$, we know the equation should be $x = -4$ since -4 is the x-value from the point.

$\boldsymbol{x = -4}$

Example 13:

Write the slope-intercept form of an equation for the line that passes through $(2, \ 5)$ and is parallel to the graph of $2x - y = 1$.

In order to find the slope of a line parallel, we need to know the slope of this line. So, let's solve for y.

$2x - y = 1$

$-2x \quad -2x$

$-y = -2x + 1$

$\frac{-y}{-1} - \frac{-2x}{-1} + \frac{1}{-1}$

$y = 2x - 1$

We know that the slope of a line parallel to this line would be the same as the slope of this line. So, since the slope of this line is 2, so is the slope of our line.

**Note: The y-intercept of the line given to use tells us where that line crosses the y-axis. We don't want our line to cross the y-axis at the same place as this line. If it did, it would be right on top of this line. We want our line to never intersect this line.

$y = mx + b$

$y = 2x + b$ *I'm only replacing slope because I need to find where the new line would cross the y-axis.

We use the point $(2, 5)$ to replace the x- and y-values.

$5 = 2(2) + b$

$5 = 4 + b$

$-4 - 4$

$1 = b$

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$\boldsymbol{y = 2x + 1}$

Example 14:

Write the slope-intercept form of an equation for the line that passes through $(7, -1)$ and is perpendicular to the graph of $x = 2$.

$x = 2$ is the graph of a vertical line. We know that a perpendicular line to a vertical line would be a horizontal line. So, our graph will have the equation of a horizontal line which is $y = \#$. Since we know that the line passes through $(7, -1)$, we know the equation should be $y = -1$ since -1 is the y-value from the point.

$\boldsymbol{y = -1}$

Example 15:

Write the slope-intercept form of an equation for the line that passes through $(-4,\ 1)$ and is perpendicular to the graph of $4x - 3y = -15$.

In order to find the slope of a line perpendicular, we need to know the slope of this line. So, let's solve for y.

$4x - 3y = -15$

$-4x \qquad -4x$

$-3y = -4x - 15$

$\frac{-3y}{-3} - \frac{-4x}{-3} - \frac{15}{-3}$

$y = \frac{4}{3}x + 5$

We know that perpendicular lines have negative reciprocal slopes. The slope of the line given to us is $\frac{4}{3}$. Therefore, the slope of a line parallel to the given line would be $-\frac{3}{4}$.

**Note: The y-intercept of the line given to use tells us where that line crosses the y-axis. We don't necessarily want our line to cross the y-axis at the same place as this line. It could, but we don't want to make that assumption.

$y = mx + b$

$y = -\frac{3}{4}x + b$ *I'm only replacing slope because I need to find where the new line would cross the y-axis.

We use the point $(-4, 1)$ to replace the x- and y-values.

$1 = -\frac{3}{4}(-4) + b$

$1 = 3 + b$

$-3 - 3$

$-2 = b$

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$$\boldsymbol{y = -\frac{3}{4}x - 2}$$

Example 16:

Write the slope-intercept form of an equation for the line that passes through $(1,\ 0)$ and is parallel to the graph of $y = \frac{1}{3}x + 2$.

We know that the slope of a line parallel to this line would be the same as the slope of this line. So, since the slope of this line is $\frac{1}{3}$, so is the slope of our line.

**Note: The y-intercept of the line given to use tells us where that line crosses the y-axis. We don't want our line to cross the y-axis at the same place as this line. If it did, it would be right on top of this line. We want our line to never intersect this line.

$y = mx + b$

$y = \frac{1}{3}x + b$ *I'm only replacing slope because I need to find where the new line would cross the y-axis.

We use the point $(1, 0)$ to replace the x- and y-values.

$0 = \frac{1}{3}(1) + b$

$0 = \frac{1}{3} + b$

$-\frac{1}{3}\ -\frac{1}{3}$

$-\frac{1}{3} = b$

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$$\boldsymbol{y = \frac{1}{3}x - \frac{1}{3}}$$

Example 17:

Write the slope-intercept form of an equation for the line that passes through $(-5,\ 1)$ and is perpendicular to the graph of $y = -\frac{5}{2}x - 2$.

We know that perpendicular lines have negative reciprocal slopes. The slope of the line given to us is $-\frac{5}{2}$. Therefore, the slope of a line parallel to the given line would be $\frac{2}{5}$.

**Note: The y-intercept of the line given to use tells us where that line crosses the y-axis. We don't necessarily want our line to cross the y-axis at the same place as this line. It could, but we don't want to make that assumption.

$y = mx + b$

$y = \frac{2}{5}x + b$ *I'm only replacing slope because I need to find where the new line would cross the y-axis.

We use the point $(-5, 1)$ to replace the x- and y-values.

$1 = \frac{2}{5}(-5) + b$

$1 = -2 + b$

$+2 + 2$

$3 = b$

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$\boldsymbol{y = \frac{2}{5}x + 3}$

Example 18:

Write the slope-intercept form of an equation for the line that passes through $(-2,\ 0)$ and is perpendicular to the graph of $y = -5$.

$y = -5$ is the graph of a horizontal line. We know that a perpendicular line to a horizontal line would be a vertical line. So, our graph will have the equation of a vertical line which is $x = \#$. Since we know that the line passes through $(-2, 0)$, we know the equation should be $x = -2$ since -2 is the x-value from the point.

$\boldsymbol{x = -2}$

Example 19:

Write the slope-intercept form of an equation for the line that passes through $(1, -3)$ and is parallel to the graph of $y = 7$.

$y = 7$ is the graph of a horizontal line. We know that a parallel line to a horizontal line would also be a horizontal line. So, our graph will have the equation of a horizontal line which is $y = \#$. Since we know that the line passes through $(1, -3)$ we know the equation should be $y = -3$ since -3 is the y-value from the point.

$\boldsymbol{y = -3}$

Example 20:

Write the slope-intercept form of an equation for the line that passes through $(-2, 1)$ and is perpendicular to the graph of $2x + y = 5$.

In order to find the slope of a line perpendicular, we need to know the slope of this line. So, let's solve for y.

$2x + y = 5$

$-2x \quad -2x$

$y = -2x + 5$

We know that perpendicular lines have negative reciprocal slopes. The slope of the line given to us is -2. Therefore, the slope of a line parallel to the given line would be $\frac{1}{2}$.

**Note: The y-intercept of the line given to use tells us where that line crosses the y-axis. We don't necessarily want our line to cross the y-axis at the same place as this line. It could, but we don't want to make that assumption.

$y = mx + b$

$y = \frac{1}{2}x + b$ *I'm only replacing slope because I need to find where the new line would cross the y-axis.

We use the point $(-2, 1)$ to replace the x- and y-values.

$1 = \frac{1}{2}(-2) + b$

$1 = -1 + b$

$1 = -1 + b$

$+1 \ +1$

$2 = b$

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$$\boldsymbol{y = \frac{1}{2}x + 2}$$