Order of Operations Notes

The **order of operations** is the rule that lets you know which operations to perform first in numerical expressions.

- Grouping symbols (Parentheses) ( ) or [ ]
- Powers (Exponents) x^n
- Multiply and Divide $\cdot \quad \div$

  from left to right
- Add and Subtract $+ \quad -$

  from left to right

Example 1: Evaluate Expressions

$15 \div 3 \cdot 6 - 4^2$	Since there are no parentheses or brackets, we will evaluate the exponent first. **Remember that 4^2 means $4 \cdot 4$ ***not*** $4 \cdot 2$.
$15 \div 3 \cdot 6 - 16$	Evaluate multiplication and division *as they occur from left to right*. That means that we may complete division before multiplication if it occurs first as we move from left to right through the expression. In this case, the $15 \div 3$ is the left-most operation involving multiplication or division so we will evaluate that first.
$5 \cdot 6 - 16$	$5 \cdot 6$ now becomes the left-most operation involving multiplication or division, so we evaluate that.
$30 - 16$	Since subtraction is the only operation left, we will evaluate the subtraction.
14	**This is our solution.

Example 2: Evaluate Expressions

$2[5 + (30 \div 6)^2]$	When multiple grouping symbols exist, start with the innermost set, in this case the $30 \div 6$.

$2[5+5^2]$	Now, we evaluate the remaining grouping symbol using the order of operations on the operations inside the grouping symbol. So, we will calculate 5^2
$2[5+25]$	The only operation remaining inside the grouping symbol is addition.
$2[30]$	When no operation is explicitly stated between a number and a grouping symbol, it indicates multiplication. So, $2[30] = 2 \cdot 30$.
60	**This is our solution.

Example 3: Evaluate Expressions Involving Substitution

$a^2 - (b^3 - 4c)$ where $a = 7, b = 3$, and $c = 5$

	The first step is always to substitute numbers in for variables. **Remember that a number and letter that a right next to each other with no operation between them is an implied multiplication
$7^2 - (3^3 - 4 \cdot 5)$	Evaluate the expression inside the parentheses using the order of operations. The exponent needs to be evaluated first. **Remember that 3^3 means $3 \cdot 3 \cdot 3$ ***not*** $3 \cdot 3$.
$7^2 - (27 - 4 \cdot 5)$	The next operation in our order of operations I multiplication/division. So, we will evaluate the multiplication.
$7^2 - (27 - 20)$	The only operation remaining in the parentheses is subtraction, so that can be evaluated.
$7^2 - 7$	Now that the grouping symbols have been evaluated, we can start at evaluating exponents again.
$49 - 7$	Since subtraction is the only operation left, we will evaluate the subtraction.
42	**This is our solution.

Example 4: Evaluate an Expression

$30 - 14 \div 2$	Evaluate division first.
$30 - 7$	Evaluate subtraction.
23	

Example 5: Evaluate an Expression

$5 \cdot 5 - 1 \cdot 3$	Evaluate left-most multiplication first.
$25 - 1 \cdot 3$	Evaluate remaining multiplication.
$25 - 3$	Evaluate subtraction.
22	

Example 6: Evaluate an Expression

$6^2 + 8 \cdot 3 + 7$	Evaluate exponent.
$36 + 8 \cdot 3 + 7$	Evaluate multiplication.
$36 + 24 + 7$	Evaluate left-most addition first.
$60 + 7$	Evaluate addition.
67	

Example 7: Evaluate an Expression

$(4 + 6)7$	Evaluate addition inside parentheses.
$(10)7$	Evaluate multiplication. (Remember that no explicit operation between a number and a parenthesis indicates multiplication).
70	

Example 8: Evaluate an Expression

$50 - (15 + 9)$	Evaluate addition inside parentheses.
$50 - 24$	Evaluate subtraction.
26	

Example 9: Evaluate an Expression

$[8(2) - 4^2] + 7(4)$	Evaluate exponent inside grouping symbols.
$[8(2) - 16] + 7(4)$	Evaluate multiplication inside grouping symbols.
$[16 - 16] + 7(4)$	Evaluate subtraction inside grouping symbols.
$0 + 7(4)$	Evaluate multiplication.
$0 + 28$	Evaluate addition.
28	

Example 10: Evaluate an Expression

$\frac{11-8}{1+7\cdot 2}$	**In a problem like this, we will evaluate the top and bottom separately. So, I will start with the numerator and then evaluate the denominator.
	Evaluate subtraction in numerator.
$\frac{3}{1+7\cdot 2}$	Evaluate multiplication in denominator.
$\frac{3}{1+14}$	Evaluate addition in denominator.
$\frac{3}{15}$	Simplify fraction.
$\frac{1}{5}$	**We know the answer will be fractional because the denominator is larger than the numerator.

Example 11: Evaluate an Expression

$\frac{(4\cdot 3)^2}{9+3}$	Evaluate multiplication in grouping symbols in numerator.
$\frac{12^2}{9+3}$	Evaluate exponent in numerator.
$\frac{144}{9+3}$	Evaluate addition in denominator.
$\frac{144}{12}$	Divide numerator and denominator.
12	**We can divide these because the numerator is larger than the denominator.

Example 12: Evaluate an Expression

$\frac{3+2^3}{5^2(6)}$	Evaluate exponent in numerator.
$\frac{3+8}{5^2(6)}$	Evaluate addition in numerator.
$\frac{11}{5^2(6)}$	Evaluate exponent in denominator.
$\frac{11}{25(6)}$	Evaluate multiplication in denominator.
$\frac{11}{150}$	**We cannot simplify this fraction.

Example 13: Evaluate an Expression Involving Substitution

$8b - a$ where $a = 4, b = 6$, and $c = 8$

	The first step is always to substitute numbers in for variables. **Remember that a number and letter that a right next to each other with no operation between them is an implied multiplication
$8(6) - 4$	Evaluate multiplication.
$48 - 4$	Evaluate subtraction.
44	

Example 14: Evaluate an Expression Involving Substitution

$2a + (b^2 \div 3)$ where $a = 4, b = 6$, and $c = 8$

	Substitute numbers in for variables.
$2(4) + (6^2 \div 3)$	Evaluate exponent inside grouping symbols.
$2(4) + (36 \div 3)$	Evaluate division inside grouping symbols.
$2(4) + 12$	Evaluate multiplication.
$8 + 12$	Evaluate addition.
20	

Example 15: Evaluate an Expression Involving Substitution

$\frac{b(9-c)}{a^2}$ where $a = 4, b = 6$, and $c = 8$

Substitute numbers in for variables.

$\frac{6(9-8)}{4^2}$ Evaluate subtraction inside grouping symbols in numerator.

$\frac{6(1)}{4^2}$ Evaluate multiplication in numerator.

$\frac{6}{4^2}$ Evaluate exponent in denominator

$\frac{6}{16}$ Simplify fraction.

$\frac{3}{8}$