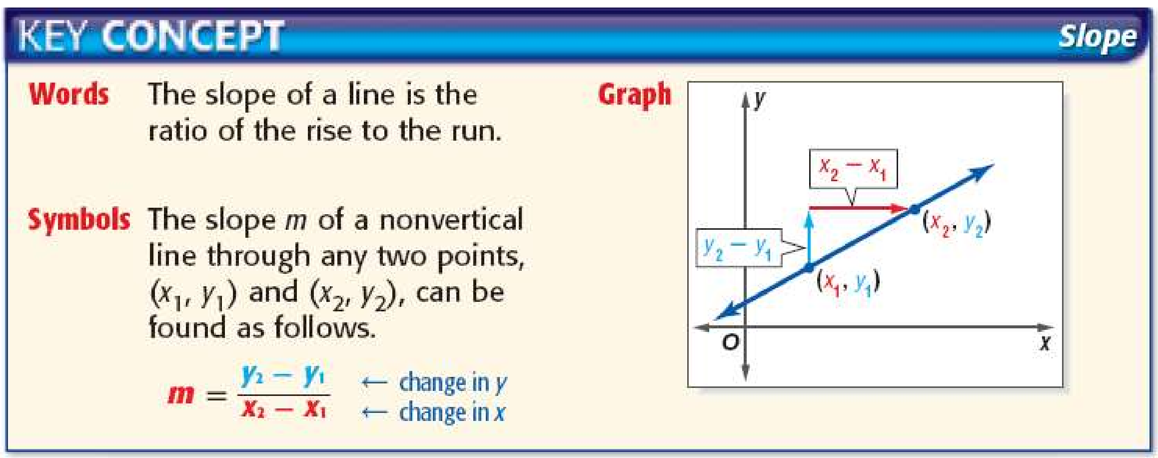
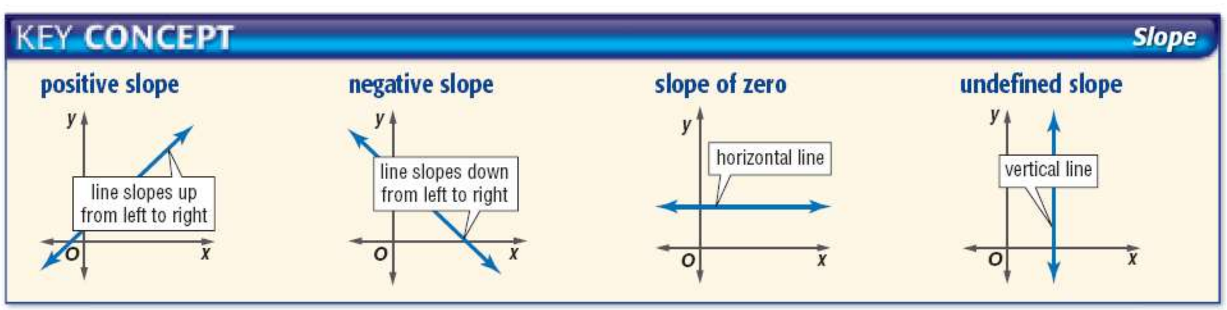
Finding Slope

Slope:

The slope of a line is the ratio of the change in the y-coordinates (rise) to the change in the x-coordinates (run).



For simplicity, I will use the delta symbol (Δ) to represent "change in." So, $\frac{change\ in\ y}{chang\ \ in\ x} = \frac{\Delta y}{\Delta x}$.



Example 1:

Find the slope of the line that passes through each pair of points. Then state whether the line rises, falls, is horizontal, or is vertical. Graph your points to check your answer.

$(-4, 9), (1, 6)$

There are two methods for calculating the slope. I will present both and you can choose the one that works best for you.

Option #1:

We know that all coordinates are in the form (x, y). So, we will split the points into an x, y table.

x	y
-4	9
1	6

From here we should find our changes. So, for the x-coordinates, we will ask ourselves

"How far is it from -4 to 1?" Or "What do I have to add into -4 to get to 1?" The answer is that we need to add five.

$+5$

x	y
-4	9
1	6

For the y-coordinates, we will ask ourselves "How far is it from 9 to 6?" Or "What do I have to add into 9 to get to 6?" The answer is that we need to add negative three.

$+5$ $\quad$ -3

x	y
-4	9
1	6

Now that we have our changes, we can calculate slope, which is $\frac{\Delta y}{\Delta x} = \frac{-3}{+5} = -\frac{3}{5}$

Option #2:

Let's start by give the coordinates a name. Putting a subscript on the letter just names the coordinate as point 1 or point 2.

$(-4, 9), (1, 6)$

$x_1, y_1 \quad x_2, y_2$

Now, we can plug these values into our slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{6 - 9}{1 - -4}$$

$$m = \frac{6 - 9}{1 + 4}$$

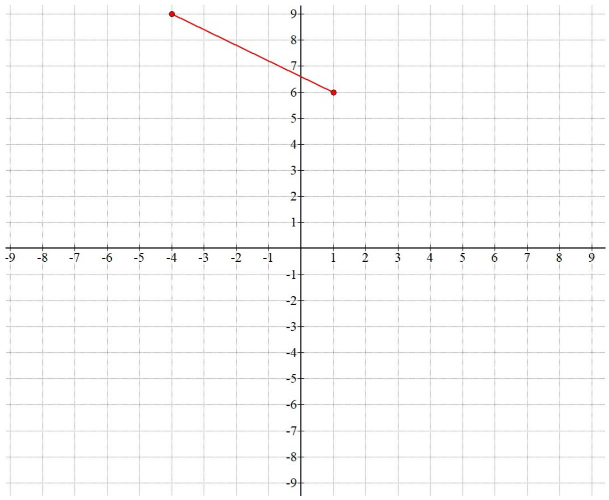
$$m = \frac{-3}{5}$$

$$m = -\frac{3}{5}$$

Either method gives us the same answer. I prefer the first method because it allows us to make fewer sign errors. However, you choose whatever works best for you.

The slope is $-\frac{3}{5}$.

According to the sign of the slope, the graph should be falling as we read it from left to right, but to double-check we will graph the points and connect them with a line.



We can see that our line is, in fact, **falling.**

Example 2:

Find the slope of the line that passes through each pair of points. Then state whether the line rises, falls, is horizontal, or is vertical. Graph your points to check your answer.

$(-4,-1),(-2,-5)$

There are two methods for calculating the slope. I will present both and you can choose the one that works best for you.

Option #1:

We know that all coordinates are in the form (x,y). So, we will split the points into an x,y table.

x	y
-4	-1
-2	-5

From here we should find our changes. So, for the x-coordinates, we will ask ourselves "How far is it from -4 to -2?" Or "What do I have to add into -4 to get to -2?" The answer is that we need to add two.

	x	y
	-4	-1
$+2$	-2	-5

Option #2:

Let's start by give the coordinates a name. Putting a subscript on the letter just names the coordinate as point 1 or point 2.

$(-4,-1),(-2,-5)$

$x_1,y_1 \qquad x_2,y_2$

Now, we can plug these values into our slope formula.

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{-5--1}{-2--4}$$

For the y-coordinates, we will ask ourselves "How far is it from -1 to -5?" Or "What do I have to add into -1 to get to -5?" The answer is that we need to add a negative four.

	x	y	
$+2$	-4	-1	-4
	-2	-5	

Now that we have our changes, we can calculate slope, which is $\frac{\Delta y}{\Delta x} = \frac{-4}{+2} = -2$

$$m = \frac{-5+1}{-2+4}$$

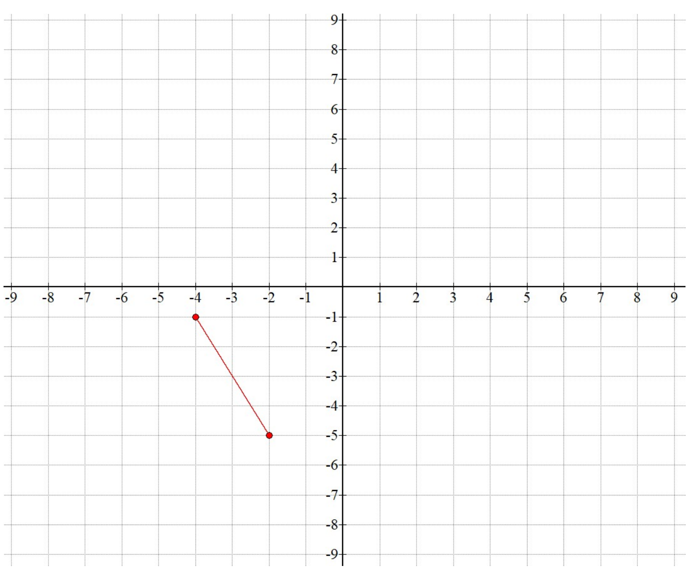
$$m = \frac{-4}{+2}$$

$$m = -2$$

Either method gives us the same answer. I prefer the first method because it allows us to make fewer sign errors. However, you choose whatever works best for you.

The slope is -2.

According to the sign of the slope, the graph should be falling as we read it from left to right, but to double-check we will graph the points and connect them with a line.



We can see that our line is, in fact, **falling.**

Example 3:

Find the slope of the line that passes through each pair of points. Then state whether the line rises, falls, is horizontal, or is vertical. Graph your points to check your answer.

$(-4, -1), (-4, -5)$

There are two methods for calculating the slope. I will present both and you can choose the one that works best for you.

Option #1:

Let's start by splitting the points into an x, y table.

x	y
-4	-1
-4	-5

From here we should find our changes. Look back at the previous problems if you don't remember the questions to ask.

	x	y	
$+0$	-4	-1	-4
	-4	-5	

Now that we have our changes, we can calculate slope, which is $\frac{\Delta y}{\Delta x} = \frac{-4}{0} =$ undefined.

We are not allowed to divide by zero. It doesn't make sense so we say this is undefined.

Option #2:

Let's start by give the coordinates a name. Putting a subscript on the letter just names the coordinate as point 1 or point 2.

$(-4, -1), (-4, -5)$

$x_1, y_1 \qquad x_2, y_2$

Now, we can plug these values into our slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-5 - -1}{-4 - -4}$$

$$m = \frac{-5 + 1}{-4 + 4}$$

$$m = \frac{-4}{0}$$

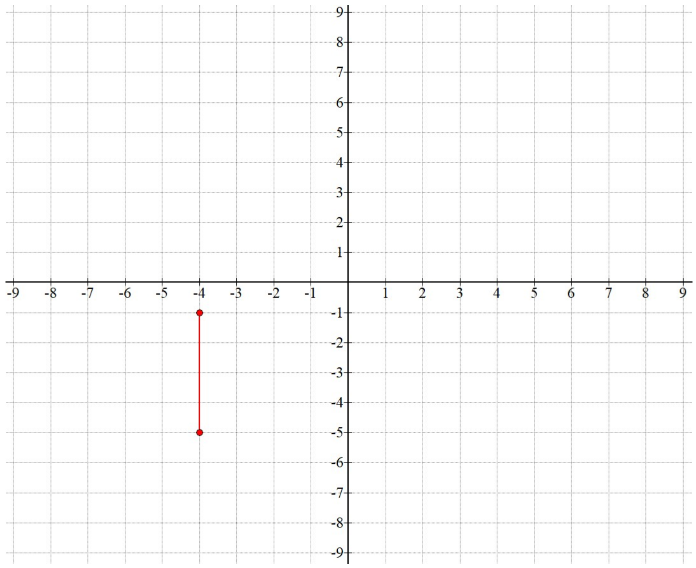
$m =$ undefined

We are not allowed to divide by zero. It doesn't make sense so we say this is undefined.

Either method gives us the same answer.

The slope is undefined.

According to our calculations for slope, this should be a vertical line, but let's graph to make sure.



We can see that our line is, in fact, **vertical.**

Example 4:

Find the slope of the line that passes through each pair of points. Then state whether the line rises, falls, is horizontal, or is vertical. Graph your points to check your answer.

$(2,1),(8,9)$

There are two methods for calculating the slope. I will present both and you can choose the one that works best for you.

Option #1:

Let's start by splitting the points into an x, y table.

x	y
2	1
8	9

From here we should find our changes. Look back at the first two problems if you don't remember the questions to ask.

	x	y	
$+6$	2	1	$+8$
	8	9	

Now that we have our changes, we can calculate slope, which is $\frac{\Delta y}{\Delta x} = \frac{8}{6} = \frac{4}{3}$.

Option #2:

Let's start by give the coordinates a name. Putting a subscript on the letter just names the coordinate as point 1 or point 2.

$(2,1),(8,9)$

$x_1, y_1 \quad x_2, y_2$

Now, we can plug these values into our slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

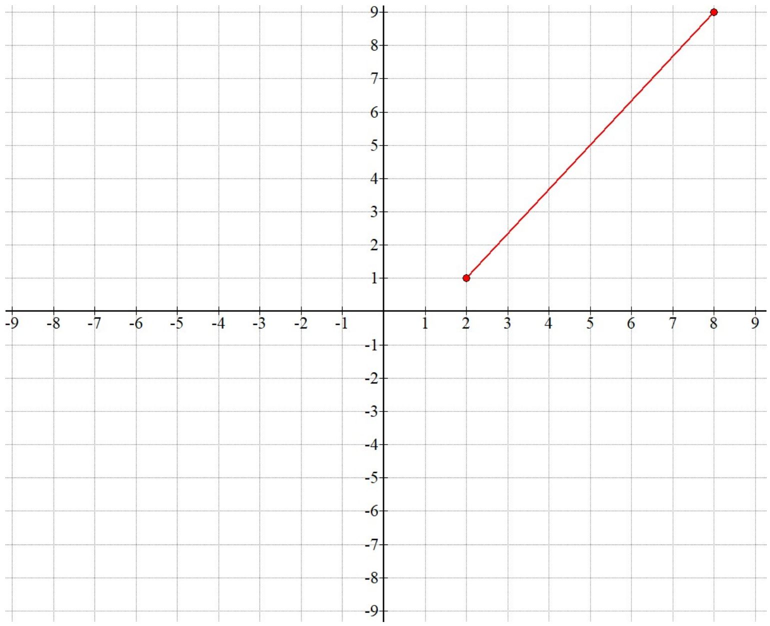
$$m = \frac{9-1}{8-2}$$

$$m = \frac{8}{6} = \frac{4}{3}$$

Either method gives us the same answer.

The slope is $\frac{4}{3}$.

According to our calculations for slope, this should be a rising line, but let's graph to make sure.



We can see that our line is, in fact, **rising.**

Example 5:

Find the slope of the line that passes through each pair of points. Then state whether the line rises, falls, is horizontal, or is vertical. Graph your points to check your answer.

$(2, 5), (6, 2)$

Option #1:

Let's start by splitting the points into an x, y table.

x	y
2	5
6	2

Option #2:

Let's start by give the coordinates a name.

$(2, 5), (6, 2)$

$x_1, y_1 \quad x_2, y_2$

From here we should find our changes. Look back at the first two problems if you don't remember the questions to ask.

$$+4 \begin{array}{|c|c|}\hline x & y \\ \hline 2 & 5 \\ \hline 6 & 2 \\ \hline \end{array} -3$$

Now that we have our changes, we can calculate slope, which is $\frac{\Delta y}{\Delta x} = \frac{-3}{4} = -\frac{3}{4}$.

Now, we can plug these values into our slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

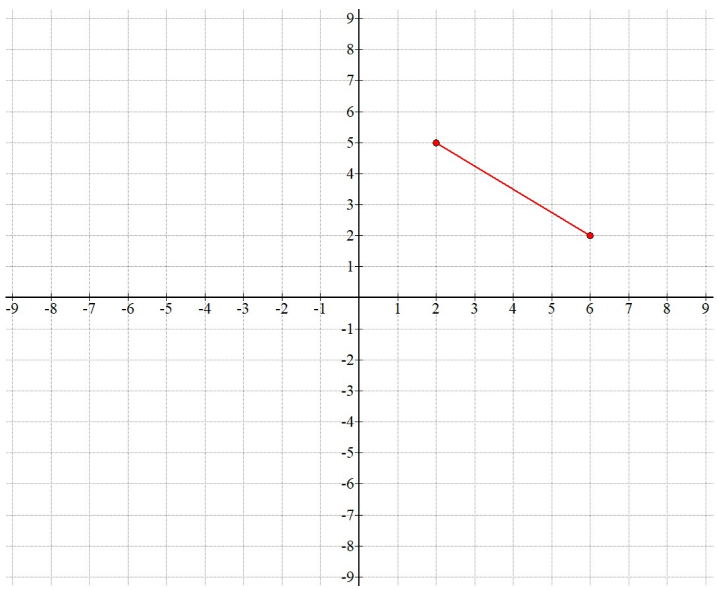
$$m = \frac{2-5}{6-2}$$

$$m = \frac{-3}{4} = -\frac{3}{4}$$

Either method gives us the same answer.

The slope is $-\frac{3}{4}$.

According to our calculations for slope, this should be a falling line, but let's graph to make sure.



We can see that our line is, in fact, **falling.**

Example 6:

Find the slope of the line that passes through each pair of points. Then state whether the line rises, falls, is horizontal, or is vertical. Graph your points to check your answer.

$(1, -2), (6, 2)$

Option #1:

Option #2:

Let's start by give the coordinates a name.

Let's start by splitting the points into an x, y table.

x	y
1	-2
6	2

From here we should find our changes. Look back at the first two problems if you don't remember the questions to ask.

	x	y	
$+5$	1	-2	$+4$
	6	2	

Now that we have our changes, we can calculate slope, which is $\frac{\Delta y}{\Delta x} = \frac{4}{5}$.

$(1, -2), (6, 2)$

$x_1, y_1 \quad x_2, y_2$

Now, we can plug these values into our slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

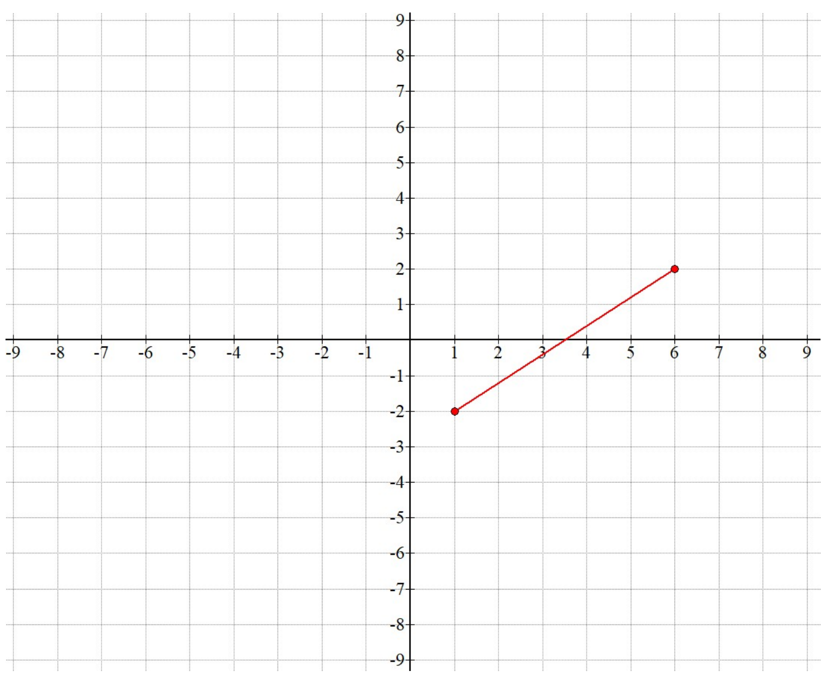
$$m = \frac{2 - -2}{6 - 1}$$

$$m = \frac{4}{5}$$

Either method gives us the same answer.

The slope is $\frac{4}{5}$.

According to our calculations for slope, this should be a rising line, but let's graph to make sure.



We can see that our line is, in fact, **rising.**

Example 7:

Find the slope of the line that passes through each pair of points. Then state whether the line rises, falls, is horizontal, or is vertical. Graph your points to check your answer.

$(4,-3),(8,-3)$

There are two methods for calculating the slope. I will present both and you can choose the one that works best for you.

Option #1:

Let's start by splitting the points into an x, y table.

x	y
4	-3
8	-3

From here we should find our changes. Look back at the first two problems if you don't remember the questions to ask.

	x	y	
$+4$	4	-3	$+0$
	8	-3	

Now that we have our changes, we can calculate slope, which is $\frac{\Delta y}{\Delta x}=\frac{0}{4}=0$.

Option #2:

Let's start by give the coordinates a name.

$(4,-3),(8,-3)$

$x_1, y_1 \quad x_2, y_2$

Now, we can plug these values into our slope formula.

$$m=\frac{y_2-y_1}{x_2-x_1}$$

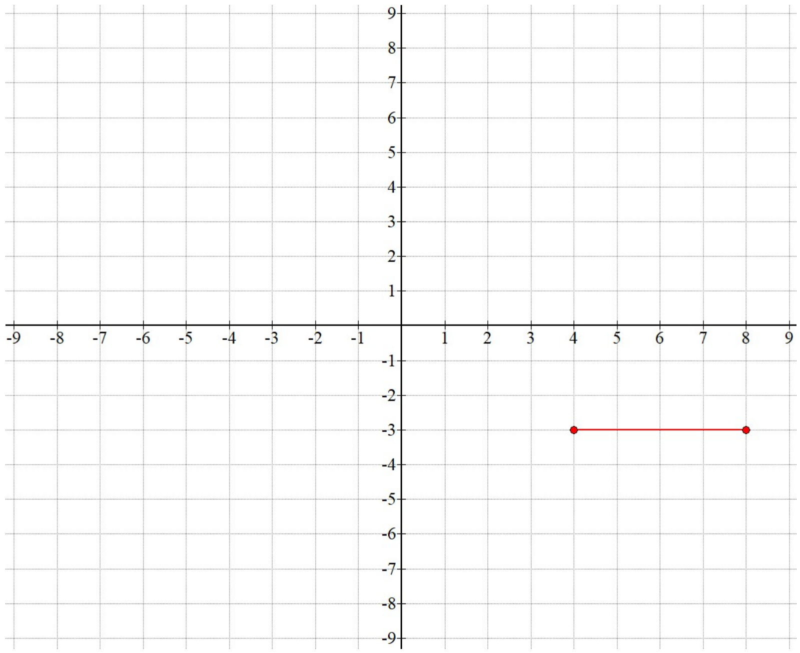
$$m=\frac{-3--3}{8-4}$$

$$m=\frac{0}{4}=0$$

Either method gives us the same answer.

The slope is 0.

According to our calculations for slope, this should be a horizontal line, but let's graph to make sure.



We can see that our line is, in fact, **horizontal.**

Example 8:

Find the slope of the line that passes through each pair of points. Then state whether the line rises, falls, is horizontal, or is vertical. Graph your points to check your answer.

$$\left(\frac{5}{3}, \frac{1}{3}\right), \left(-\frac{1}{3}, \frac{8}{3}\right)$$

There are two methods for calculating the slope. I will present both and you can choose the one that works best for you.

Option #1:

Let's start by splitting the points into an x, y table.

x	y
$\frac{5}{3}$	$\frac{1}{3}$
$-\frac{1}{3}$	$\frac{8}{3}$

From here we should find our changes. Look back at the first two problems if you don't remember the questions to ask.

Option #2:

Let's start by give the coordinates a name.

$$\left(\frac{5}{3}, \frac{1}{3}\right), \left(-\frac{1}{3}, \frac{8}{3}\right)$$

$$x_1, y_1 \quad x_2, y_2$$

Now, we can plug these values into our slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$-\frac{6}{3} = -2$

x	y
$\frac{5}{3}$	$\frac{1}{3}$
$-\frac{1}{3}$	$\frac{8}{3}$

$+\frac{7}{3}$

Now that we have our changes, we can calculate slope, which is $\frac{\Delta y}{\Delta x} = \frac{\frac{7}{3}}{-2} = \frac{7}{3} \cdot -\frac{1}{2} = -\frac{7}{6}$.

$$m = \frac{\frac{8}{3} - \frac{1}{3}}{-\frac{1}{3} - \frac{5}{3}}$$

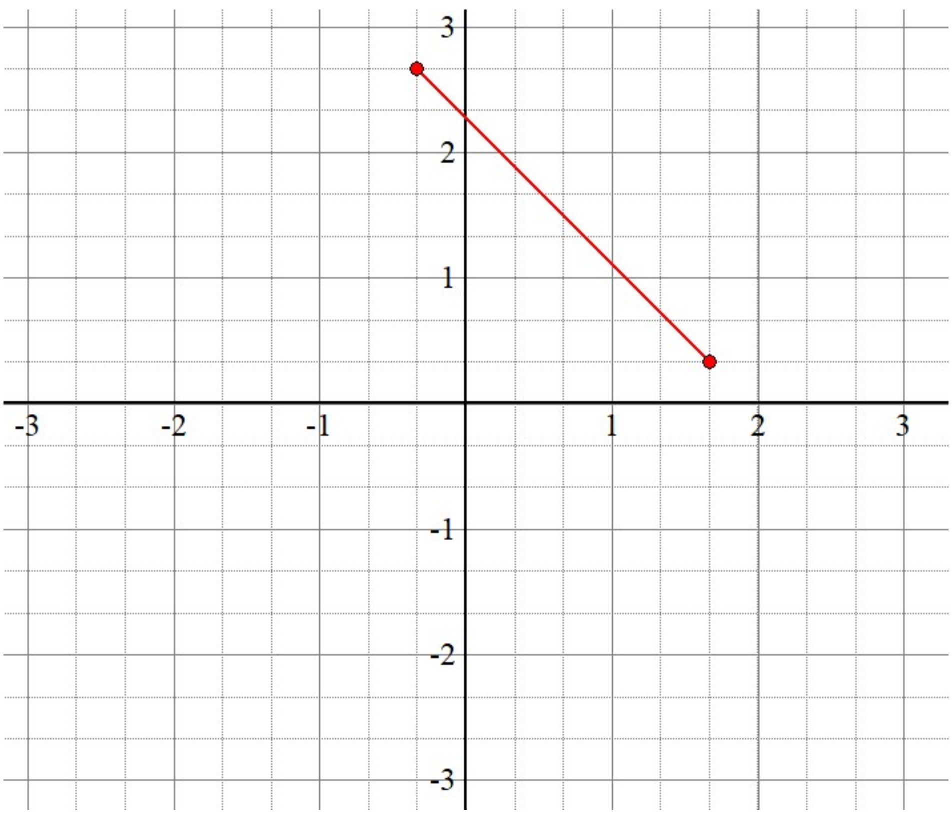
$$m = \frac{\frac{7}{3}}{-\frac{6}{3}} = \frac{7}{3} \cdot -\frac{3}{6} = -\frac{21}{18} = -\frac{7}{6}$$

Either method gives us the same answer.

The slope is $-\frac{7}{6}$.

According to our calculations for slope, this should be a falling line, but let's graph to make sure.

The easiest way to graph fractions is to change the scale so that each grid mark is $\frac{1}{3}$, which means every 3rd grid line is a whole unit.



We can see that our line is, in fact, **falling.**

Example 9:

Find the slope of the line that passes through each pair of points. Then state whether the line rises, falls, is horizontal, or is vertical. Graph your points to check your answer.

$\left(-\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{2},-\frac{1}{2}\right)$

Option #1:

Let's start by splitting the points into an x, y table.

x	y
$-\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{2}$

From here we should find our changes. Look back at the first two problems if you don't remember the questions to ask.

$+\frac{2}{2}=1$

x	y
$-\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$-\frac{1}{2}$

$-\frac{2}{2}=-1$

Now that we have our changes, we can calculate slope, which is $\frac{\Delta y}{\Delta x}=\frac{-1}{1}=-1$

Option #2:

Let's start by give the coordinates a name.

$\left(-\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{2},-\frac{1}{2}\right)$

$x_1, y_1 \quad x_2, y_2$

Now, we can plug these values into our slope formula.

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{-\frac{1}{2}-\frac{1}{2}}{\frac{1}{2}--\frac{1}{2}}$$

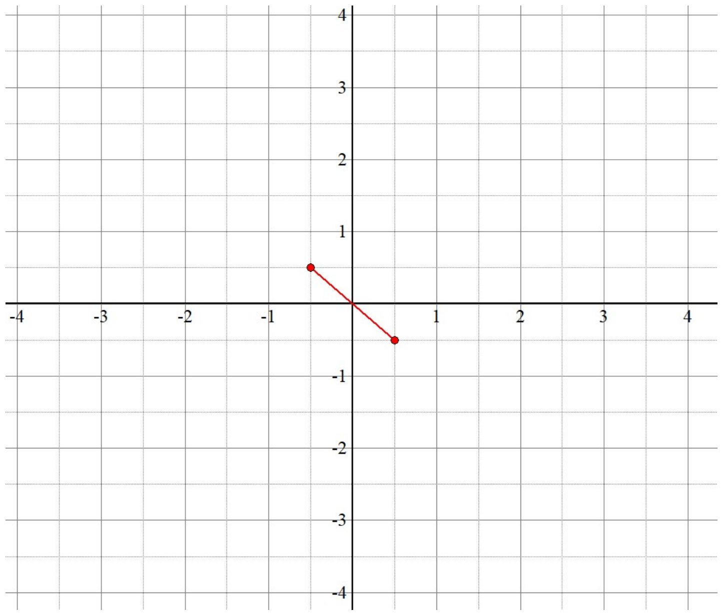
$$m=\frac{-\frac{2}{2}}{\frac{2}{2}}=\frac{-1}{1}=-1$$

Either method gives us the same answer.

The slope is -1.

According to our calculations for slope, this should be a falling line, but let's graph to make sure.

The easiest way to graph fractions is to change the scale so that each grid mark is $\frac{1}{2}$, which means every 2nd grid line is a whole unit.



We can see that our line is, in fact, **falling.**

<u>Example 10:</u>

Find the slope of the line that passes through each pair of points. Then state whether the line rises, falls, is horizontal, or is vertical. Graph your points to check your answer.

$(0.1, -0.2), (0.6, 0.2)$

Option #1:

Let's start by splitting the points into an x, y table.

x	y
0.1	-0.2
0.6	0.2

From here we should find our changes. Look back at the first two problems if you don't remember the questions to ask.

	x	y	
	0.1	-0.2	
$+0.5$	0.6	0.2	$+0.4$

Now that we have our changes, we can calculate slope, which is $\frac{\Delta y}{\Delta x} = \frac{0.4}{0.5} = \frac{4}{5}$

Option #2:

Let's start by give the coordinates a name.

$(0.1, -0.2), (0.6, 0.2)$

$x_1, \ y_1 \qquad x_2, \ y_2$

Now, we can plug these values into our slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

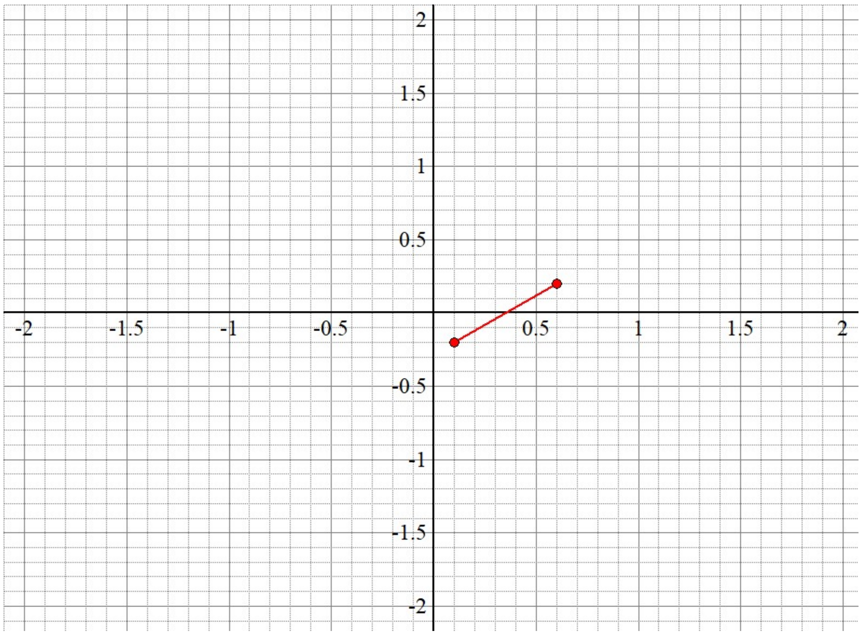
$$m = \frac{0.2 - -0.2}{0.6 - 0.1}$$

$$m = \frac{0.4}{0.5} = \frac{4}{5}$$

The slope is $\frac{4}{5}$.

According to our calculations for slope, this should be a rising line, but let's graph to make sure.

The easiest way to graph decimals in tenths is to change the scale so that each grid mark is 0.1, which means every 10^{th} grid line is a whole unit.



We can see that our line is, in fact, **rising.**

Example 11:

Find the slope of the line that passes through each pair of points. Then state whether the line rises, falls, is horizontal, or is vertical. Graph your points to check your answer.

$(-0.4, 0.5), (-0.8, -0.5)$

Option #1:

Let's start by splitting the points into an x, y table.

x	y
-0.4	0.5
-0.8	-0.5

From here we should find our changes. Look back at the first two problems if you don't remember the questions to ask.

Option #2:

Let's start by give the coordinates a name.

$(-0.4, 0.5), (-0.8, -0.5)$

$x_1, \; y_1 \qquad x_2, \; y_2$

Now, we can plug these values into our slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

	x	y	
-0.4	-0.4	0.5	-1
	-0.8	-0.5	

Now that we have our changes, we can calculate slope, which is $\frac{\Delta y}{\Delta x} = \frac{-1}{-0.4} = \frac{10}{4} = \frac{5}{2}$

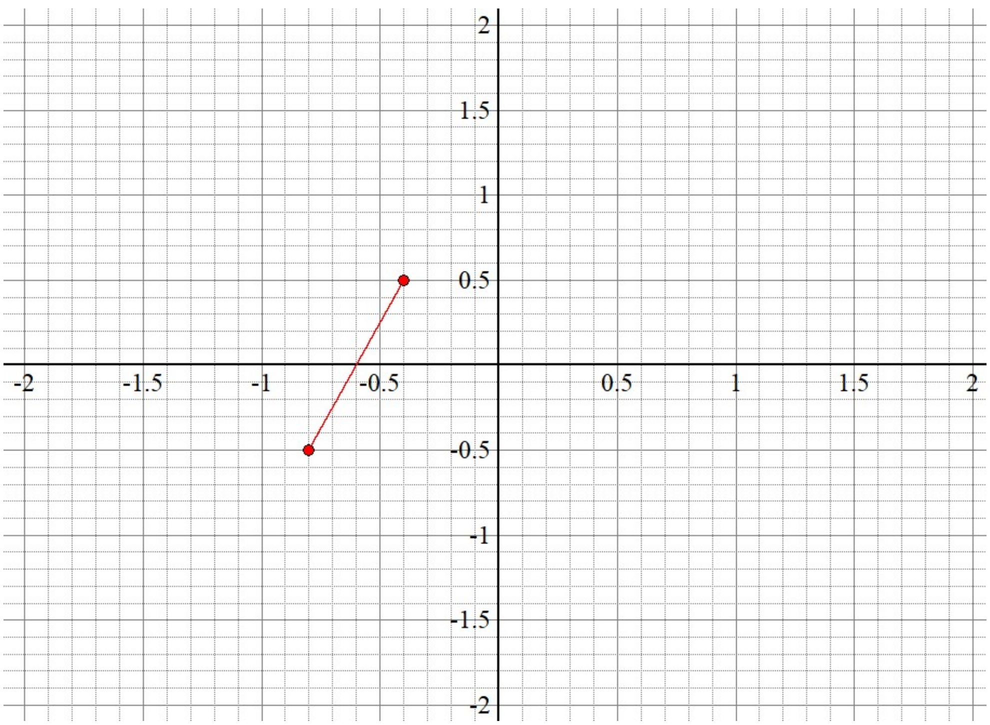
$$m = \frac{-0.5 - 0.5}{-0.8 - -0.4}$$

$$m = \frac{-1}{-0.4} = \frac{10}{4} = \frac{5}{2}$$

The slope is $\frac{5}{2}$.

According to our calculations for slope, this should be a rising line, but let's graph to make sure.

The easiest way to graph decimals in tenths is to change the scale so that each grid mark is 0.1, which means every 10^{th} grid line is a whole unit.



We can see that our line is, in fact, **rising.**