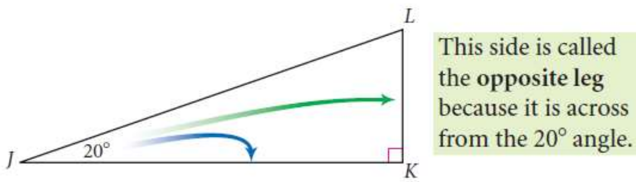
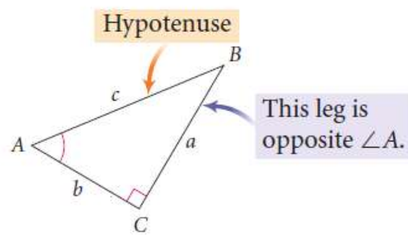
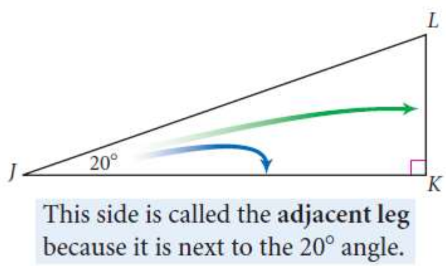
Lesson 12.1 – Trigonometric Ratios

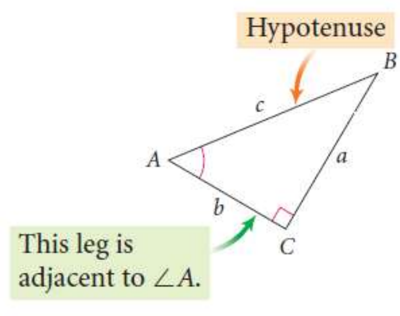
Opposite Leg - The leg of the triangle directly opposite the selected angle is referred to as the opposite leg.



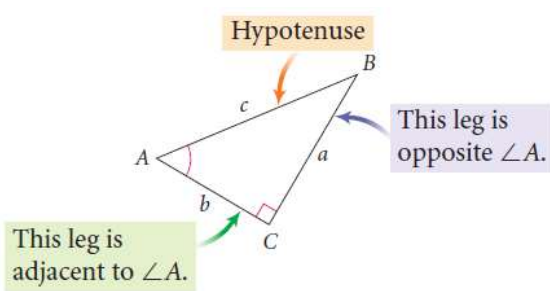


Adjacent Leg - The leg of the triangle next to the selected angle (that is not the hypotenuse) is referred to as the adjacent leg.





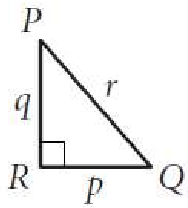
Trigonometric Ratios



For any acute angle A in a right triangle:	In $\triangle ABC$ above:
sine of $\angle A = \dfrac{\text{length of opposite leg}}{\text{length of hypotenuse}}$	$\sin A = \dfrac{a}{c}$
cosine of $\angle A = \dfrac{\text{length of adjacent leg}}{\text{length of hypotenuse}}$	$\cos A = \dfrac{b}{c}$
tangent of $\angle A = \dfrac{\text{length of opposite leg}}{\text{length of adjacent leg}}$	$\tan A = \dfrac{a}{b}$

Example 1: Give the answer as a fraction in terms of p, q, and r.

$\sin P =$________

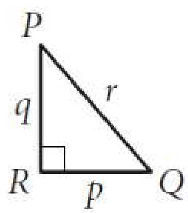


$$\sin P = \frac{\text{opposite leg to } P}{\text{hypotenuse}}$$

$$\mathbf{sin}\,\boldsymbol{P} = \boldsymbol{\frac{p}{r}}$$

Example 2: Give the answer as a fraction in terms of p, q, and r.

$\cos P =$________

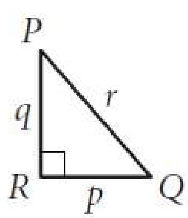


$$\cos P = \frac{\text{adjacent leg to } P}{\text{hypotenuse}}$$

$$\mathbf{cos}\,\boldsymbol{P} = \boldsymbol{\frac{q}{r}}$$

Example 3: Give the answer as a fraction in terms of p, q, and r.

$\tan P =$________

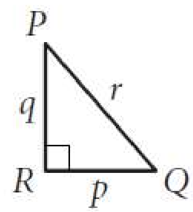


$$\tan P = \frac{\text{opposite leg to } P}{\text{adjagent leg to } P}$$

$$\mathbf{tan}\,\boldsymbol{P} = \boldsymbol{\frac{p}{q}}$$

Example 4: Give the answer as a fraction in terms of p, q, and r.

$\sin Q =$________

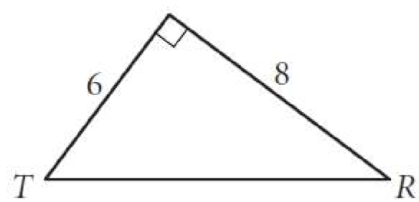


$$\sin Q = \frac{\text{opposite leg to } Q}{\text{hypotenuse}}$$

$$\boldsymbol{\sin Q = \frac{q}{r}}$$

Example 5: Give the answer as fraction in simplest form.

$\sin T =$________



We need to start by finding the length of the hypotenuse $\overline{TR}$. We will use the Pythagorean Theorem for this.

$a^2 + b^2 = c^2$

$6^2 + 8^2 = c^2$

$36 + 64 = c^2$

$100 = c^2$

$\sqrt{100} = \sqrt{c^2}$

$10 = c$

$TR = 10$

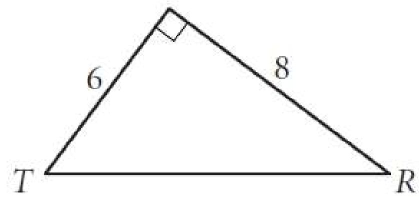
$$\sin T = \frac{\text{opposite leg to } T}{\text{hypotenuse}}$$

$$\sin T = \frac{8}{10} = \frac{4}{5}$$

$$\boldsymbol{\sin T = \frac{4}{5}}$$

Example 6: Give the answer as fraction in simplest form.

$\cos T =$________



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$a^2 + b^2 = c^2$

$6^2 + 8^2 = c^2$

$36 + 64 = c^2$

$100 = c^2$

$\sqrt{100} = \sqrt{c^2}$

$10 = c$

$TR = 10$

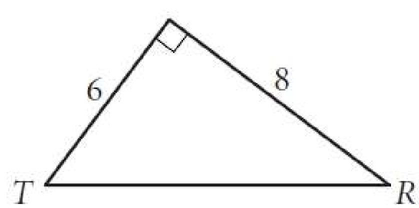
$$\cos T = \frac{\text{adjacent leg to } T}{\text{hypotenuse}}$$

$$\cos T = \frac{6}{10} = \frac{3}{5}$$

$$\boldsymbol{\cos T = \frac{3}{5}}$$

Example 7: Give the answer as fraction in simplest form.

$\tan T =$________



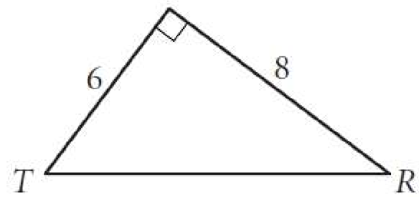
$$\tan T = \frac{\text{opposite leg to } T}{\text{adjacent leg to } T}$$

$$\tan T = \frac{8}{6} = \frac{4}{3}$$

$$\boldsymbol{\tan T = \frac{4}{3}}$$

Example 8: Give the answer as fraction in simplest form.

$\sin R =$________



We need to start by finding the length of the hypotenuse $\overline{TR}$. We will use the Pythagorean Theorem for this.

$a^2 + b^2 = c^2$

$6^2 + 8^2 = c^2$

$36 + 64 = c^2$

$100 = c^2$

$\sqrt{100} = \sqrt{c^2}$

$10 = c$

$TR = 10$

$$\sin R = \frac{\text{opposite leg to } R}{\text{hypotenuse}}$$

$$\sin R = \frac{6}{10} = \frac{3}{5}$$

$\boldsymbol{\sin R = \frac{3}{5}}$\

Example 9: Solve for x. Express each answer accurate to the nearest 0.01.

$$\cos 64° = \frac{x}{28}$$

$$28 \cdot \cos 64° = \frac{x}{28} \cdot 28$$

$28 \cdot \cos 64° = x$

$12.27 = x$

$\boldsymbol{x \approx 12.27}$

Example 10: Solve for x. Express each answer accurate to the nearest 0.01.

$$\sin 24° = \frac{12.1}{x}$$

$$x \cdot \sin 24° = \frac{12.1}{x} \cdot x$$

$$x \cdot \sin 24° = 12.1$$

$$\frac{x \cdot \sin 24°}{\sin 24°} = \frac{12.1}{\sin 24°}$$

$$x = \frac{12.1}{\sin 24°}$$

$$x = 29.75$$

$$\boldsymbol{x \approx 29.75}$$

<u>Example 11: Solve for x. Express each answer accurate to the nearest 0.01.</u>

$$\tan 51° = \frac{x}{14.8}$$

$$14.8 \cdot \tan 51° = \frac{x}{14.8} \cdot 14.8$$

$$14.8 \cdot \tan 51° = x$$

$$18.28 = x$$

$$\boldsymbol{x \approx 18.28}$$

<u>Example 12: Find the measure of each angle to the nearest degree.</u>

$$\sin A = 0.9455$$

$$\sin^{-1} \sin A = \sin^{-1} 0.9455$$

$$A = \sin^{-1} 0.9455$$

$$A = 70.9967 \ldots$$

$$\boldsymbol{A = 71°}$$

<u>Example 13: Find the measure of each angle to the nearest degree.</u>

$$\tan B = \frac{4}{3}$$

$$\tan^{-1} \tan B = \tan^{-1} \frac{4}{3}$$

$B = \tan^{-1}\frac{4}{3}$

$B = 53.1301 \ldots$

$\boldsymbol{B = 53°}$

<u>Example 14: Find the measure of each angle to the nearest degree.</u>

$\cos C = 0.8660$

$\cos^{-1}\cos C = \cos^{-1} 0.8660$

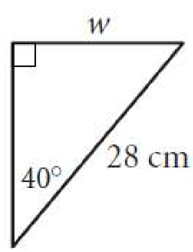
$C = \cos^{-1} 0.8660$

$C = 30.0029 \ldots$

$\boldsymbol{C = 30°}$

<u>Example 14: Write a trigonometric equation you can use to solve for the unknown value. Then find the value to the nearest 0.1.</u>

$w \approx$__________



We always want to use the trigonometric function that uses ratios of sides we have relative to a known angle that is not the right angle. In this case, that is the 40° angle.

Relative to the 40° angle, we know the hypotenuse and we are looking for the opposite side.

The trigonometric ratio that compares opposite side and hypotenuse is sine.

$\sin 40° = \frac{w}{28}$

$28 \cdot \sin 40° = \frac{w}{28} \cdot 28$

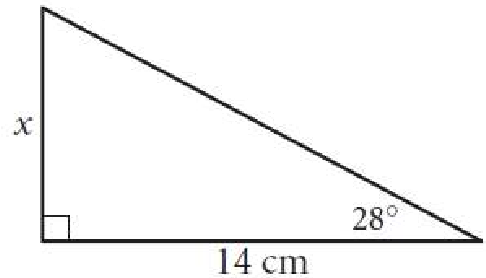
$28 \cdot \sin 40° = w$

$17.99805 \ldots = w$

$\boldsymbol{w \approx 18.0}$ **cm**

Example 15: Write a trigonometric equation you can use to solve for the unknown value. Then find the value to the nearest 0.1.

$x \approx$_________



We always want to use the trigonometric function that uses ratios of sides we have relative to a known angle that is not the right angle. In this case, that is the 28° angle.

Relative to the 28° angle, we know the adjacent side and we are looking for the opposite side.

The trigonometric ratio that compares opposite side and adjacent side is tangent.

$$\tan 28° = \frac{x}{14}$$

$$14 \cdot \tan 28° = \frac{x}{14} \cdot 14$$

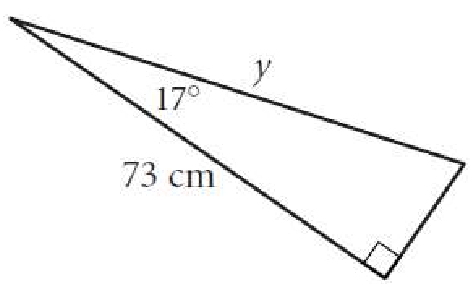
$$14 \cdot \tan 28° = x$$

$$7.443932 \ldots = x$$

$$\boldsymbol{x \approx 7.4} \textbf{ cm}$$

Example 16: Write a trigonometric equation you can use to solve for the unknown value. Then find the value to the nearest 0.1.

$y \approx$_________



We always want to use the trigonometric function that uses ratios of sides we have relative to a known angle that is not the right angle. In this case, that is the 17° angle.

Relative to the 17° angle, we know the adjacent side and we are looking for the hypotenuse.

The trigonometric ratio that compares hypotenuse and adjacent side is cosine.

$$\cos 17° = \frac{73}{y}$$

$$y \cdot \cos 17° = \frac{73}{y} \cdot y$$

$$y \cdot \cos 17° = 73$$

$$\frac{y \cdot \cos 17°}{\cos 17°} = \frac{73}{\cos 17°}$$

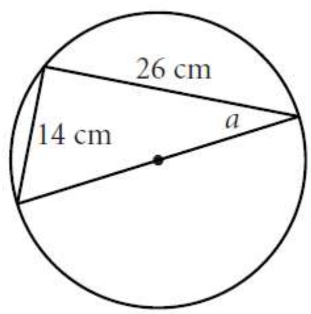
$$y = \frac{73}{\cos 17°}$$

$$y = 76.335498 \ldots$$

$$\boldsymbol{y \approx 76.3\ \mathbf{cm}}$$

<u>Example 18: Find the value of each unknown to the nearest degree.</u>

$a \approx$__________



We always want to use the trigonometric function that uses ratios of sides we have relative to the angle we want to find that is not the right angle. In this case, that is a.

Relative to a, we know the adjacent side and the opposite side.

The trigonometric ratio that compares adjacent side to opposite side is tangent.

$$\tan a = \frac{14}{26}$$

$$\tan^{-1} \tan a = \tan^{-1} \frac{14}{26}$$

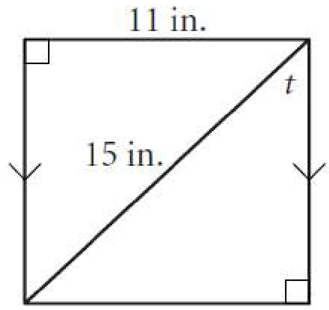
$$a = \tan^{-1} \frac{14}{26}$$

$$a = 28.30075 \ldots$$

$$\boldsymbol{a \approx 28°}$$

Example 19: Find the value of each unknown to the nearest degree.

$t \approx$_________



We always want to use the trigonometric function that uses ratios of sides we have relative to the angle we want to find that is not the right angle. In this case, that is t.

Relative to t, we know the opposite side and the hypotenuse. The opposite side will be 11 in because the figure is a rectangle and opposite sides are congruent.

The trigonometric ratio that compares opposite side to hypotenuse is sine.

$\sin t = \frac{11}{15}$

$\sin^{-1} \sin t = \sin^{-1} \frac{11}{15}$

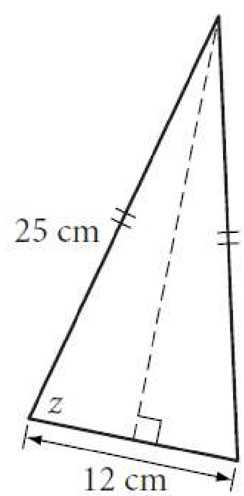
$t = \sin^{-1} \frac{11}{15}$

$t = 47.1657 \ldots$

$\boldsymbol{t \approx 47°}$

Example 20: Find the value of each unknown to the nearest degree.

$z \approx$_________



We always want to use the trigonometric function that uses ratios of sides we have relative to the angle we want to find that is not the right angle. In this case, that is z.

Relative to z, we know the adjacent side and the hypotenuse. The adjacent side will be 6 cm. The triangle is isosceles and we know that the altitude will bisect the opposite side.

The trigonometric ratio that compares adjacent side to hypotenuse is cosine.

$$\cos z = \frac{6}{25}$$

$$\cos^{-1}\cos z = \cos^{-1}\frac{6}{25}$$

$$z = \cos^{-1}\frac{6}{25}$$

$$z = 76.11345\ldots$$

$$\boldsymbol{z \approx 76}^\circ$$