Lesson 10.7 – Solving Volume Problems

Example 1:

A triangular pyramid has a volume of 180 cm^3 and a height of 12 cm. Find the length of a side of the triangular base if the triangle's height from that side is 6 cm.

$V = \frac{1}{3}BH$ **Pyramid volume formula

$180 = \frac{1}{3}(B)(12)$

$180 = 4B$

$45 = B$ **This is the area of the base, so now we need an area formula

$A = \frac{1}{2}bh$ **The base is a triangle, so we will use the triangle area formula

$45 = \frac{1}{2}(b)(6)$

$45 = 3b$

$15 = b$

The side of the triangle is 15 cm long.

Example 2:

A trapezoidal pyramid has a volume of 3168 cm^3, and its height is 36 cm. The length of the two bases of the trapezoidal base are 20 cm and 28 cm. What is the height of the trapezoidal base?

$V = \frac{1}{3}BH$ **Pyramid volume formula

$3168 = \frac{1}{3}(B)(36)$

$3168 = 12B$

$264 = B$ **This is the area of the base, so now we need an area formula

$A = \frac{1}{2}(b_1 + b_2)h$ **The base is a trapezoid, so we will use the trapezoid area formula

$264 = \frac{1}{2}(20 + 28)h$

$264 = \frac{1}{2}(48)h$

$264 = 24h$

$11 = h$

The height of the trapezoid is 11 cm.

<u>Example 3:</u>

The volume of a cylinder is 200π cm^3. Find the radius of the base if the cylinder has a height of 8 cm.

$V = BH$ **Cylinder volume formula

$200\pi = B(8)$

$25\pi = B$ **This is the area of the base, so now we need an area formula

$A = \pi r^2$ **The base is a circle, so we will use the circle area formula

$25\pi = \pi r^2$

$25 = r^2$

$\sqrt{25} = r$

$5 = r$

The radius of the cylinder is 5 cm.

<u>Example 4:</u>

A cone has volume 108π cm^3. Find the radius of the base if the cone has a height of 16 cm.

$V = \frac{1}{3}BH$ **Cone volume formula

$108\pi = \frac{1}{3}(B)(16)$

$108\pi = 5\frac{1}{3}B$

$20\frac{1}{4}\pi = B$ **This is the area of the base, so now we need an area formula

$A = \pi r^2$ **The base is a circle, so we will use the circle area formula

$$20\frac{1}{4}\pi = \pi r^2$$

$$20\frac{1}{4} = r^2$$

$$\sqrt{20\frac{1}{4}} = r$$

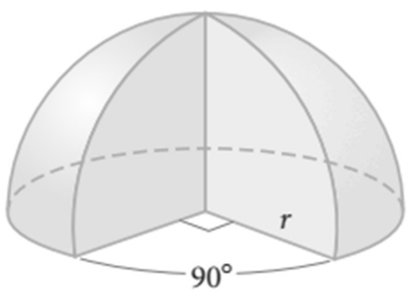
$$4.55 = r$$

The radius of the cone is $\mathbf{4.5}$ cm.

Example 5:

$V = 864\pi$ cm^3.

$r =$____________



If we had an entire sphere, the formula would be $V = \frac{4}{3}\pi r^3$. Half of a sphere (a hemisphere) would be $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$. Since we only want part of a hemisphere, we need to multiply the hemisphere formula by the number of degrees out of 360 that we want to keep.

$$V = \frac{270}{360}\left(\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)\right)$$

If we multiply all three fractions $\frac{270}{360} \cdot \frac{1}{2} \cdot \frac{4}{3}$, we get $\frac{1}{2}$. So, let's simplify our formula.

$$V = \frac{1}{2}\pi r^3$$

$$864\pi = \frac{1}{2}\pi r^3$$

$$864 = \frac{1}{2}r^3$$

$$1728 = r^3$$

$\sqrt[3]{1728} = r$ **Since we are getting rid of a power of three, we will take a third root.

$12 = r$

The radius of the slice of the sphere is $\mathbf{12}$ cm.