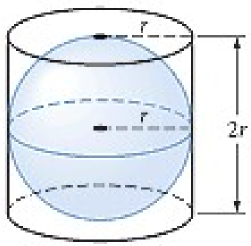
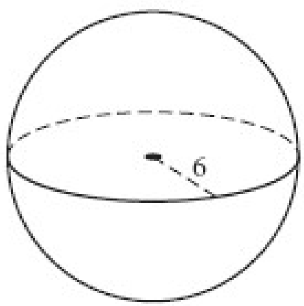
Lesson 10.6 – Volume of a Sphere

Sphere Volume Conjecture - The volume of a sphere with radius r is given by the formula $V = \frac{4}{3}\pi r^3$.



Example 1: Find the volume of the solid. All measurements are in centimeters.



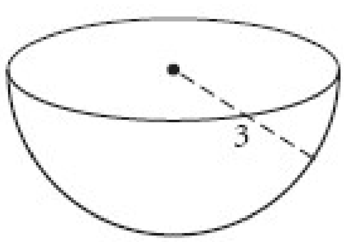
$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (6)^3$ **The radius is 6 cm.

$V = \frac{4}{3}\pi (216)$

$\boldsymbol{V = 288\pi}$ **cm³**

Example 2: Find the volume of the solid. All measurements are in centimeters.



Since this figure is a hemisphere, or half of a sphere, we will need to divide the volume by 2.

$$V = \frac{\frac{4}{3}\pi r^3}{2}$$

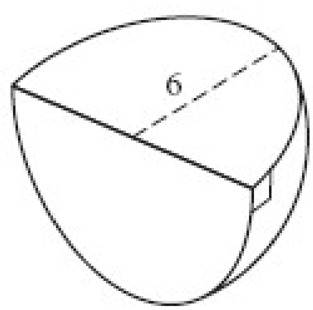
$$V = \frac{\frac{4}{3}\pi (3)^3}{2}$$

$$V = \frac{\frac{4}{3}\pi (27)}{2}$$

$$V = \frac{36\pi}{2}$$

$\boldsymbol{V = 18\pi}$ **cm³**

Example 3: Find the volume of the solid. All measurements are in centimeters.



This figure is a quarter of a sphere, so we will need to divide the volume formula by 4. Although it looks strange, the radius of this portion of a sphere is 6. If you imagine putting the other three pieces back onto the sphere, you can see that the 6 is actually a measurement from the center of the entire sphere to the outside of the sphere.

$$V = \frac{\frac{4}{3}\pi r^3}{4}$$

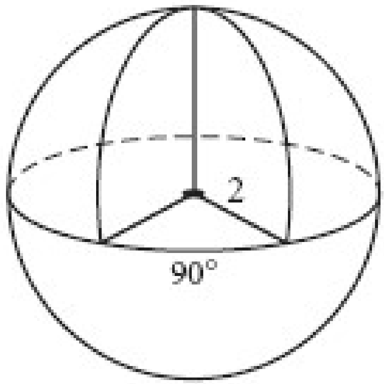
$$V = \frac{\frac{4}{3}\pi (6)^3}{4}$$

$$V = \frac{\frac{4}{3}\pi (216)}{4}$$

$$V = \frac{288\pi}{4}$$

$\boldsymbol{V = 72\pi}$ **cm³**

Example 4: Find the volume of the solid. All measurements are in centimeters.



This figure is a sphere with a slice taken out of the top hemisphere. So, we will split this sphere into two pieces and then add the volumes back together at the end.

Let's start by finding the volume of the whole sphere:

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(2)^3$ **The radius is 2 cm.

$V = \frac{4}{3}\pi(8)$

$V = 10\frac{2}{3}\pi$ cm^3 **This is the volume of the entire sphere.

Let's split this volume into its two hemispheres:

Top hemisphere:

$V = \frac{10\frac{2}{3}\pi}{2} = 5\frac{1}{3}\pi$

Bottom hemisphere:

$V = \frac{10\frac{2}{3}\pi}{2} = 5\frac{1}{3}\pi$

Since the bottom hemisphere is complete, we will keep that $5\frac{1}{3}\pi$ and add it back into what we calculate for the top portion. In the top hemisphere, we do not want the entire hemisphere. We want to remove a 90° slice. This means we want to keep a $270°$ slice,

Top portion:

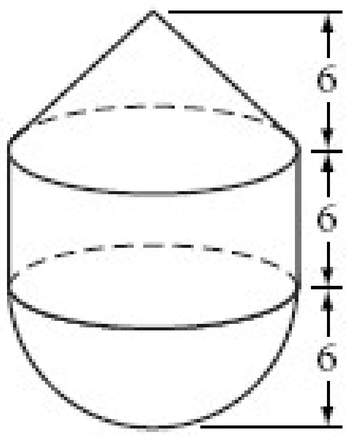
$V = \frac{270}{360}\left(5\frac{1}{3}\pi\right) = 4\pi$

Let's add the two portions (top portion and entire bottom hemisphere) back together:

$V = 4\pi + 5\frac{1}{3}\pi = 9\frac{1}{3}\pi$

$\boldsymbol{V = 9\frac{1}{3}\pi}$ **cm^3**

Example 5: Find the volume of the solid. All measurements are in centimeters.



This figure is a composite of three figures: a cone, a cylinder, and a hemisphere. We will split this figure into these three parts and add their volumes at the end.

Because the "height" of the hemisphere is also the distance from the center to the outside of the sphere, it is also the radius of the hemisphere. The radius of the hemisphere is the same as the radius of the cylinder and the cone.

Remember that to find volume of the cone and the cylinder, we will need to find area of the base first.

Cone:	Cylinder:	Hemisphere:
$A = \pi r^2$	$A = \pi r^2$	$V = \frac{\frac{4}{3}\pi r^3}{2}$
$A = \pi(6)^2$	$A = \pi(6)^2$	$V = \frac{\frac{4}{3}\pi(6)^3}{2}$
$A = 36\pi$	$A = 36\pi$	$V = \frac{\frac{4}{3}\pi(216)}{2}$
$V = \frac{1}{3}BH$	$V = BH$	$V = \frac{288\pi}{2}$
$V = \frac{1}{3}(36\pi)(6)$	$V = (36\pi)(6)$	$V = 144\pi$
$V = 72\pi$	$V = 216\pi$	

Now, let's add all three figures together:

$V = 72\pi + 216\pi + 144\pi = 432\pi$

$\boldsymbol{V = 432\pi}$ **cm³**