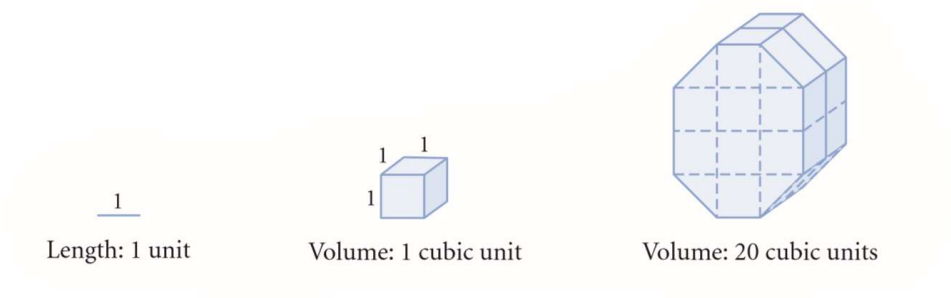
Lesson 10.2 – Volume of Prisms and Cylinders

Volume is the measure of the amount of space contained in a solid.



Prism-Cylinder Volume Conjecture:

If B is the area of the base of a prism or cylinder and H is the height of the solid, then the formula for the volume is $V = BH$.

Important Area Formulas that you will need:

Rectangle: $A = bh$

Parallelogram: $A = bh$

Triangle: $A = \frac{1}{2}bh$

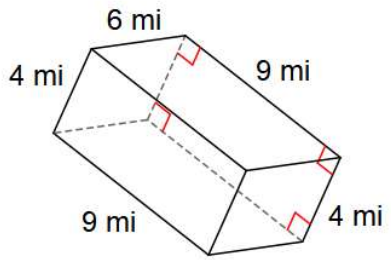
Trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Circle: $A = \pi r^2$

Sector: $A = \frac{a}{360}\pi r^2$

Regular Polygon: $A = \frac{1}{2}asn$

Example 1: Rectangular Prism Volume



The base is a rectangle. **We can call any rectangle the base in a rectangular prism. For this problem, I will use the 9 mi by 6 mi rectangle as the base. That means that the height will be the distance between those two sides that are congruent (4 mi).

$B = bh$ **This is the area formula for a rectangle. We have used B to indicate that this will be the area of the base of the prism.

$B = (9)(6)$

$B = 54$ **Now that we have area of the base, we can find the volume.

$V = BH$

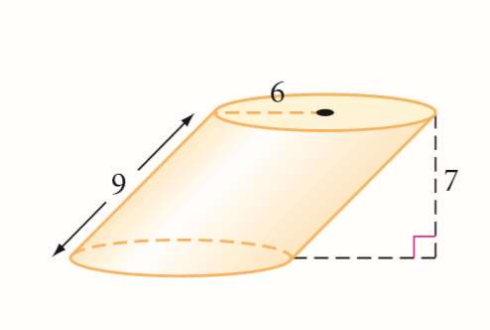
$V = (54)(4)$

$V = 216$

The volume of the rectangular prism is **216 mi^3**.

Example 2: Cylinder Volume

Find the volume of an oblique cylinder that has a base with a radius of 6 inches and a height of 7 inches.



The base is a circle.

$B = \pi r^2$

$B = \pi(6)^2$

$B = 36\pi$

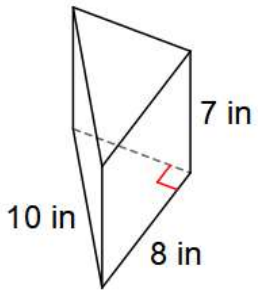
$V = BH$

$V = (36\pi)(7)$ **Remember that the height should be perpendicular to the base.

$V = 252\pi$

The volume of the cylinder is **252π in^3**, or about **791.68 in^3**.

Example 3: Right Triangular Prism Volume



The base is a right triangle. We do not know the two sides that meet at a right angle. So, we will need to use the Pythagorean Theorem to find the missing side.

$a^2 + b^2 = c^2$

$8^2 + b^2 = 10^2$

$64 + b^2 = 100$

$-64 \qquad -64$

$b^2 = 36$

$\sqrt{b^2} = \sqrt{36}$

$b = 6$ **Remember that this side and the 8 in side will meet at a right angle and therefore must be the base and height of the triangular base.

The base is a triangle.

$B = \frac{1}{2}bh$

$B = \frac{1}{2}(6)(8)$

$B = 24$

$V = BH$

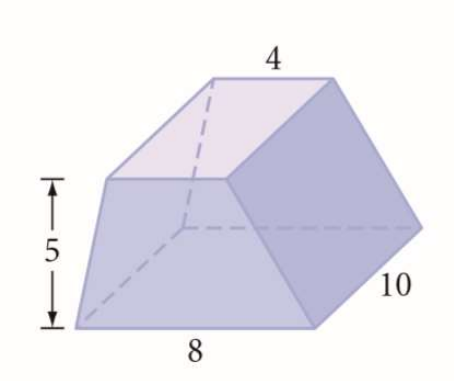
$V = (24)(7)$

$V = 168$

The volume of the right triangular prism is **168 in^3**.

Example 4: Trapezoidal Prism Volume

Find the volume of a right trapezoidal prism that has a height of 10 cm. The two bases of the trapezoid measure 4 cm and 8 cm, and its height is 5 cm.



The base is a trapezoid.

$B = \frac{1}{2}(b_1 + b_2)h$

$B = \frac{1}{2}(8 + 4)(5)$

$B = \frac{1}{2}(12)(5)$

$B = 30$

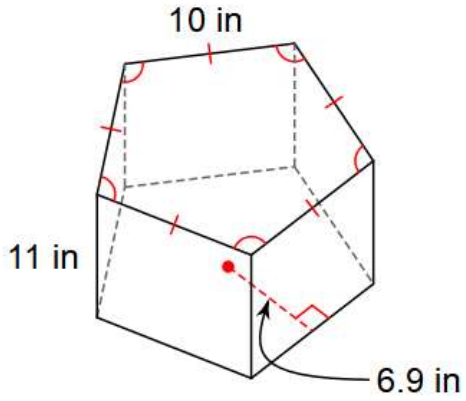
$V = BH$

$V = (30)(10)$ **The height of the prism is always the distance between the bases (in this case the trapezoids). That means that the height is 10 cm.

$V = 300$

The volume of the trapezoidal prism is **300 cm^3**.

Example 5: Regular Polygonal Prism Volume



The base is a regular polygon.

$B = \frac{1}{2}asn$

$B = \frac{1}{2}(6.9)(10)(5)$

$B = 172.5$

$V = BH$

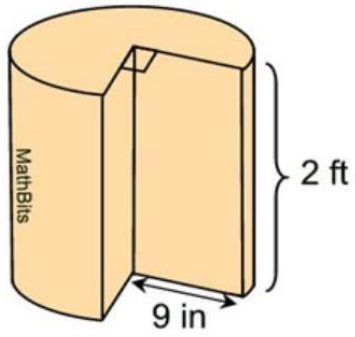
$V = (172.5)(11)$

$V = 1897.5$

The volume of the regular polygonal prism is **1897.5 in^3**.

Example 6: Cylinder with a slice removed

A 90º wedge is removed from a right circular cylinder with a radius of 9 inches and a height of 2 feet. Which choice is the volume of the remaining portion of the cylinder?



The base is a sector with arc measure 270° ($360° - 90° = 270°$).

$B = \frac{a}{360}\pi r^2$

$B = \frac{270}{360}\pi(9)^2$

$B = \frac{3}{4}\pi(81)$

$B = 60.75\pi$

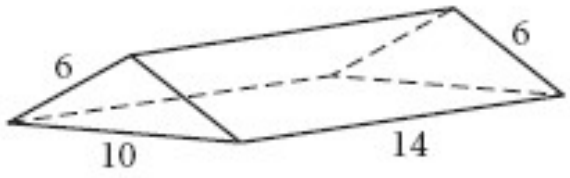
$V = BH$

$V = (60.75\pi)(24)$ **I'm using 24 as my height because I need the radius of the base and the height to have similar units. So, since there are 12 inches in a foot, there are 24 inches in 2 feet.

$V = 1458\pi$

The volume of the cylinder with the slice removed is **1458π in^3**, or about **4580.44 in^3**.

Example 7: Triangular Prism Volume



The base is an isosceles triangle. We do not know the height of the base triangle. So, we will need to use the Pythagorean Theorem to find the missing height.

$a^2 + b^2 = c^2$

$5^2 + b^2 = 6^2$

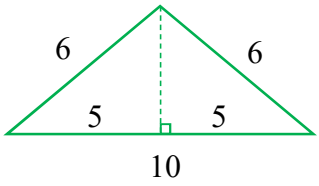
$25 + b^2 = 36$

$-25 \qquad -25$

$b^2 = 11$

$\sqrt{b^2} = \sqrt{11}$

$b \approx 3.32$ **Remember that this is the height of the triangle with base 10 units.



The base is a triangle.

$$B = \frac{1}{2}bh$$

$$B = \frac{1}{2}(10)(3.32)$$

$$B = 16.58$$

$$V = BH$$

$$V = (16.58)(14)$$

$$V = 232.16$$

The volume of the triangular prism is **232.16 units^3**.