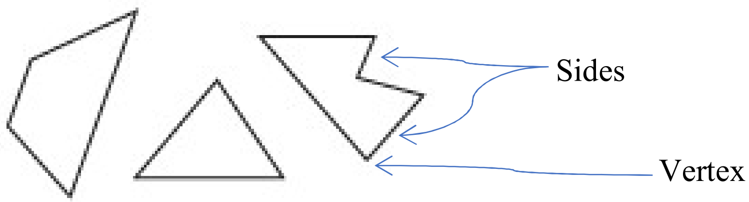
Lesson 1.4 – Polygons

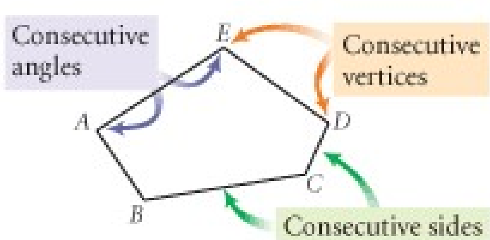
Polygon - A polygon is a closed figure in a plane formed by connecting line segments (called sides of the polygon) endpoint to endpoint (called vertices) with each line segment intersecting exactly two others.



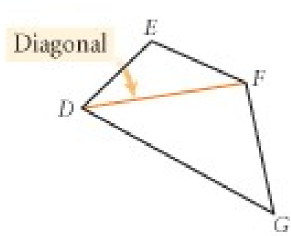
Naming Polygons:

Number of Sides	Polygon Name	Number of Sides	Polygon Name	Number of Sides	Polygon Name
3	Triangle	7	Heptagon	11	Undecagon
4	Quadrilateral	8	Octagon	12	Dodecagon
5	Pentagon	9	Nonagon	n	n −gon
6	Hexagon	10	Decagon		

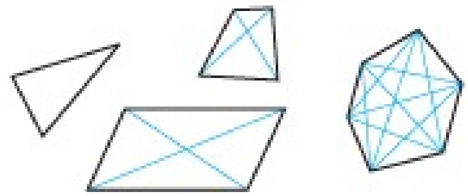
Consecutive - Angles, sides, or vertices that come one right after the other.



Diagonal - A diagonal of a polygon is a line segment that connects two nonconsecutive vertices.

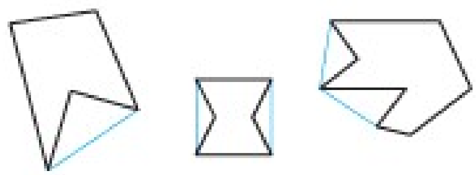


Convex - A polygon is convex if no diagonal is outside the polygon.



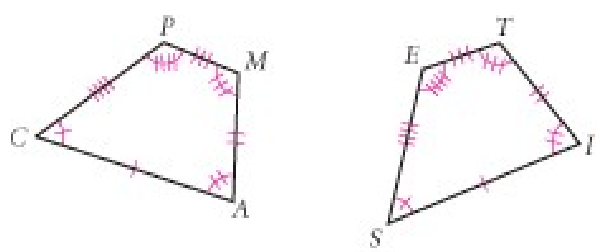
Convex polygons: All diagonals are inside

Concave - A polygon is concave if at least one diagonal is outside the polygon.



Concave polygons: One or more diagonals are outside

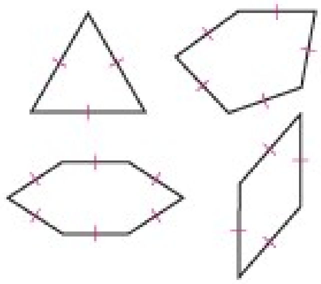
Congruent (≅) Polygons - Two polygons are congruent if and only if they are exactly the same size and shape. Naming congruent polygons and making congruence statements is order specific.



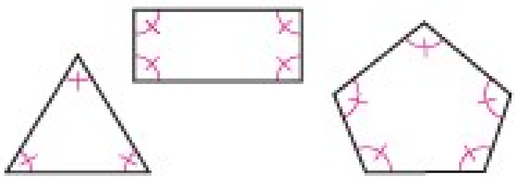
$CAMP \cong SITE$

Perimeter - The perimeter of a polygon equals the sum of the lengths of its sides. The polygon has perimeter 37 m.

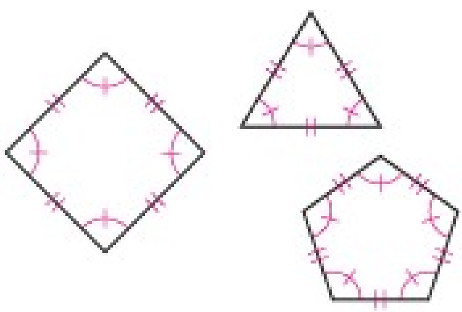
Equilateral Polygon - Equilateral polygons are polygons in which all sides have equal length.



Equiangular Polygon - Equiangular polygons are polygons in which all angles have equal measure.



Regular Polygon - Regular polygons are polygons that are both equilateral and equiangular.



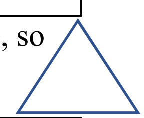
<u>Example 1: Fill in the table</u>

Polygon Name	Number of sides	Number of diagonals
1. Triangle		

We know that a triangle has 3 sides.

Polygon Name	Number of sides	Number of diagonals
1. Triangle	3	

If we look at a picture of a triangle, we can see that none of the vertices are nonconsecutive, so we cannot draw any diagonals.

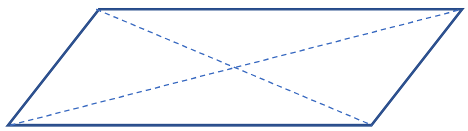


Polygon Name	Number of sides	Number of diagonals
1. Triangle	3	0

Example 2: Fill in the table

Polygon Name	Number of sides	Number of diagonals
2.		2

The last example had 3 sides, so let's try a figure with 4 sides and see if we can draw two diagonals.



Polygon Name	Number of sides	Number of diagonals
2.	4	2

We know that a 4-sided figure is called a quadrilateral.

Polygon Name	Number of sides	Number of diagonals
2. Quadrilateral	4	2

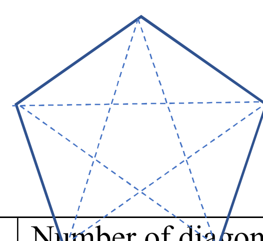
Example 3: Fill in the table

Polygon Name	Number of sides	Number of diagonals
3.	5	

We know that a 5-sided figure is called a pentagon.

Polygon Name	Number of sides	Number of diagonals
3. Pentagon	5	

Let's see how many diagonals we can draw in a pentagon…



We can draw 5 diagonals.

Polygon Name	Number of sides	Number of diagonals
3. Pentagon	5	5

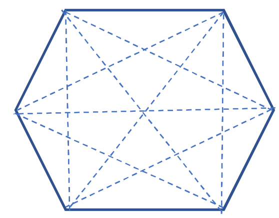
Example 4: Fill in the table

Polygon Name	Number of sides	Number of diagonals
4. Hexagon		

We know that a hexagon has 6 sides.

Polygon Name	Number of sides	Number of diagonals
4. Hexagon	6	

Let's see how many diagonals we can draw in a hexagon…



We can draw 9 diagonals.

Polygon Name	Number of sides	Number of diagonals
4. Hexagon	6	9

Example 5: Fill in the table

Polygon Name	Number of sides	Number of diagonals
5. Heptagon		

We know that a heptagon has 7 sides.

Polygon Name	Number of sides	Number of diagonals
5. Heptagon	7	

Rather than draw diagonals for this problem, let's look for a pattern in the number of sides and diagonals.

Number of Sides	3	4	5	6	7
Number of Diagonals	0	2	5	9	?

+2 +3 +4 +5

So, a heptagon has $9 + 5 = 14$ diagonals.

Polygon Name	Number of sides	Number of diagonals
5. Heptagon	7	14

Example 6: Fill in the table

Polygon Name	Number of sides	Number of diagonals
6.	8	

We know that an 8 sided figure is called an octagon.

Polygon Name	Number of sides	Number of diagonals
6. Octagon	8	

Rather than draw diagonals for this problem, let's continue to use the pattern in the number of sides and diagonals.

Number of Sides	3	4	5	6	7	8
Number of Diagonals	0	2	5	9	14	?

+2 +3 +4 +5 +6

So, an octagon has $14 + 6 = 20$ diagonals.

Polygon Name	Number of sides	Number of diagonals
6. Octagon	8	20

Example 7: Fill in the table

Polygon Name	Number of sides	Number of diagonals
7.		35

Let's use our pattern for the number of diagonals and continue to add sides until we reach 35 diagonals.

Number of Sides	3	4	5	6	7	8	9
Number of Diagonals	0	2	5	9	14	20	

+2 +3 +4 +5 +6 +7

Number of Sides	3	4	5	6	7	8	9	10
Number of Diagonals	0	2	5	9	14	20	27	

+2 +3 +4 +5 +6 +7 +8

So, the figure with 35 diagonals has 10 sides.

Polygon Name	Number of sides	Number of diagonals
7.	10	35

We know that a polygon with 10 sides is called a decagon.

Polygon Name	Number of sides	Number of diagonals
7. Decagon	10	35

Example 8: Fill in the table

Polygon Name	Number of sides	Number of diagonals
8.	12	

We know that a polygon with 12 sides is called a dodecagon.

Polygon Name	Number of sides	Number of diagonals
8. Dodecagon	12	

Let's use our pattern to determine the number of diagonals in a dodecagon.

Number of Sides	3	4	5	6	7	8	9	10	11	12
Number of Diagonals	0	2	5	9	14	20	27	35		

+2 +3 +4 +5 +6 +7 +8 +9 +10

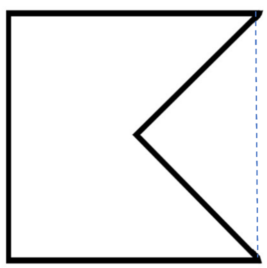
A decagon has $35 + 9 + 10 = 44$ diagonals.

Polygon Name	Number of sides	Number of diagonals
8. Dodecagon	12	44

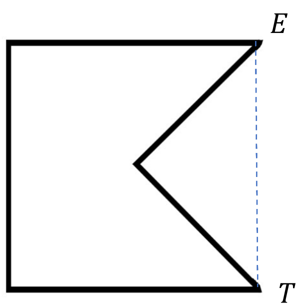
Example 9: Sketch the figure

Concave pentagon $PENTA$, with external diagonal $\overline{ET}$, and $\overline{TA} \cong \overline{PE}$.

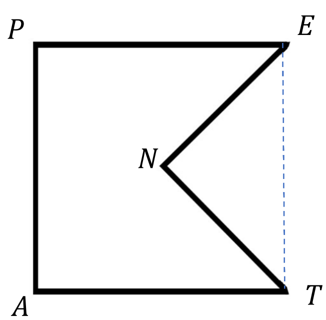
In order to have a concave figure, one diagonal must lie outside the figure.



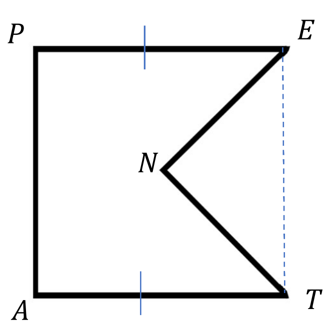
We need to name the external diagonal $\overline{ET}$.



Remember the the name of the polygon should be either clockwise or counterclockwise, so fill in the rest of the name.



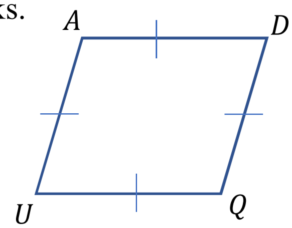
The last thing we need to do is mark $\overline{TA} \cong \overline{PE}$.



Example 10: Sketch the figure

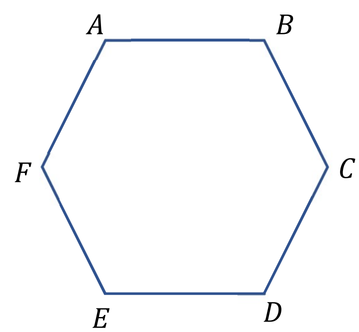
Equilateral quadrilateral $QUAD$, with $\angle Q \ncong \angle U$.

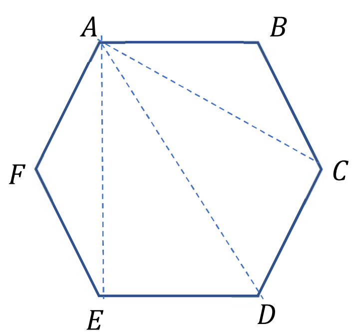
A quadrilateral has four sides. If it is equilateral, we know that all four sides have to have the same measure. To say that $\angle Q \ncong \angle U$ (the line through the congruent sign means that the angles are not congruent) means that we cannot draw a square. Make sure to mark congruent sides with tick marks.



Example 11: Use hexagon $ABCDEF$

Name the diagonals from A.



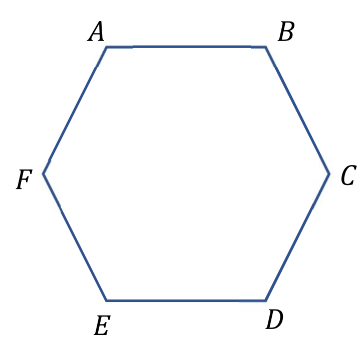


Because diagonals are line segments, we need to use the line segment marking as we name the three diagonals.

$\overline{AC}, \overline{AD}, \overline{AE}$

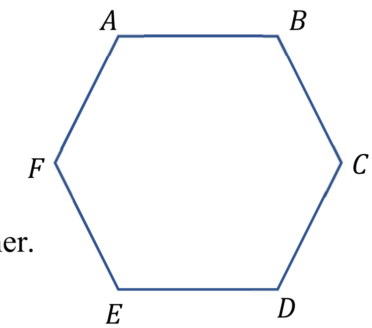
Example 12: Use hexagon $ABCDEF$

Name a pair of consecutive sides.



Consecutive sides are any two sides that come one right after the other.

If we start with side $\overline{FE}$, the side consecutive to that is either $\overline{FA}$ or $\overline{ED}$.

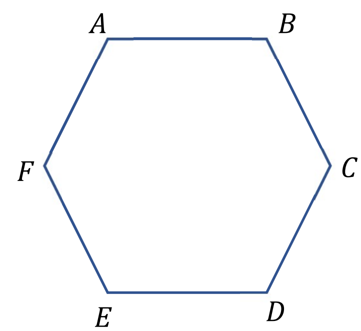


Example 13: Use hexagon $ABCDEF$

Name a pair of consecutive angles.

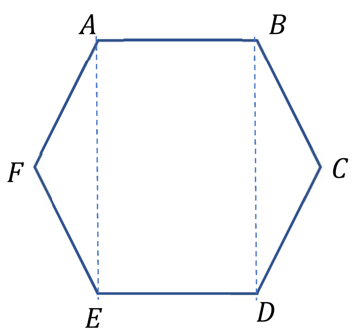
Consecutive angles are any two angles that come one right after the other.

If we start with $\angle B$, the angle consecutive to that is either $\angle A$ or $\overline{\angle C}$.



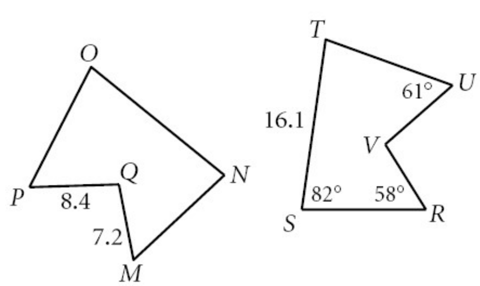
Example 14: Use hexagon $ABCDEF$

Name a pair of non-intersecting diagonals.



$\overline{AE}$ and $\overline{BD}$

Example 15: Complete the statement

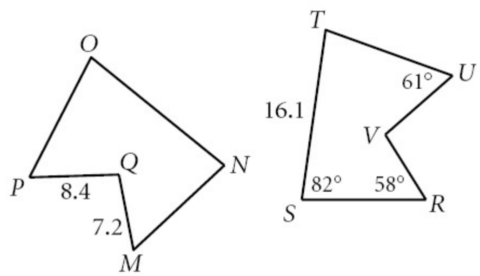


$MNOPQ \cong RSTUV$

$m\angle N =$________

Because of the lowercase m in front of the angle name, we know we are looking for the angle measure. We look at the names of the two figures to figure out which other angle $\angle N$ is congruent to. Since N and S are both the second letters of the names of the figures, $\angle N \cong \angle S$. $m\angle S = 82°$, so $m\angle N = 82°$.

Example 16: Complete the statement

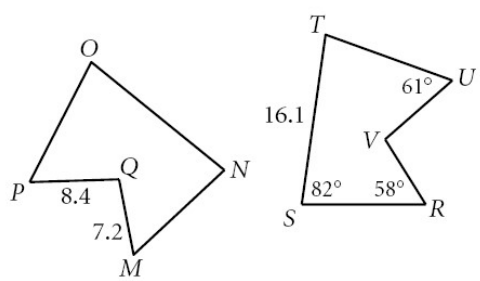


$MNOPQ \cong RSTUV$

$VR =$________

Because there is no figure over the top of the two points, we know we are looking for the segment measure. We look at the names of the two figures to figure out which other side $\overline{VR}$ is congruent to. Since V and R are the first and last letters of the name of the figure, we look at M and Q in the first figure. Therefore, $\overline{VR} \cong \overline{QM}$. $QM = 7.2$, so $VR = 7.2$.

Example 17: Complete the statement

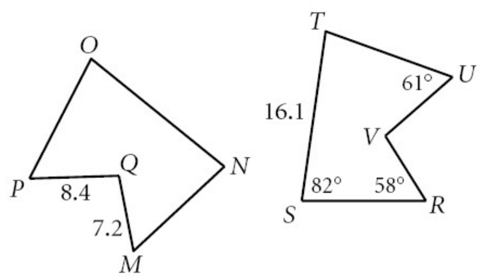


$MNOPQ \cong RSTUV$

$m\angle P =$________

Because of the lowercase m in front of the angle name, we know we are looking for the angle measure. We look at the names of the two figures to figure out which other angle $\angle P$ is congruent to. Since P and U are both the fourth letters of the names of the figures, $\angle P \cong \angle U$. $m\angle U = 61°$, so $m\angle P = 61°$.

Example 18: Complete the statement



$MNOPQ \cong RSTUV$

$ON =$________

Because there is no figure over the top of the two points, we know we are looking for the segment measure. We look at the names of the two figures to figure out which other side $\overline{ON}$ is congruent to. Since O and N are the third and fourth letters of the name of the figure, we look at S and T in the second figure. Therefore, $\overline{ON} \cong \overline{ST}$. $ST = 16.1$, so $ON = 16.1$.

Example 19: Complete

The perimeter of a regular pentagon is 31 cm. Find the length of each side.

If a polygon is regular, it means that it is both equilateral and equiangular. Since all the sides are congruent and add to 31 cm we can divide 31 cm by the number of sides. A pentagon has 5 sides.

$\frac{31}{5} = 6.2$ cm per side.