Adding and Subtracting Rational Numbers Notes

Adding and Subtracting Integers Rules:

If the signs are the same:

- The magnitude of the number should get larger.
- Add the absolute values of the numbers and keep the sign of the numbers.
- Example: $-5 + (-3) = -(5 + 3) = -8$
- Note: $-5 + (-3)$ is the same thing as $-5 - 3$

If the signs are different:

- The magnitude of the number should get smaller.
- Subtract the absolute values of the numbers and take the sign of the number with a larger magnitude.
- Example: $-5 + 3 = -(5 - 3) = -2$
- Example: $4 - 7 = -(7 - 4) = -3$
- Example: $-2 + 8 = +(8 - 2) = 6$

Two negative numbers directly together (minus a negative) should be turned into a positive.

- Example: $2 - (-7) = 2 + 7$
- Example: $-5 - (-8) = -5 + 8$

Example 1: Evaluate

$-8 + 3$

The 8 is negative and the 3 is positive.

Since the signs are opposite, we should find the difference between 8 and 3.

Since the 8 has the larger absolute value, our answer will have the same sign as 8 which is negative.

So, $-(8 - 3) = -5$

$-8 + 3 = -5$

Example 2: Evaluate

$-2+(-1)$

**The parenthesis are just here because we have two operations, $(+\,-)$, directly next to each other. You will see parentheses whenever this situation exists.

The 2 is negative and the 1 is also negative.

Since the signs are the same, we should find the sum of 2 and 1.

Since both values are negative, our answer will also be negative.

So, $-(2+1)=-3$

$-2+(-1)=-3$

Example 3: Evaluate

$6-7$

The 6 is positive and the 7 is negative. Remember that the sign of a number is the same as the sign the directly precedes the number. A subtraction is the same as a negative number.

Since the signs are opposite, we should find the difference between 6 and 7.

Since the 7 has the larger absolute value, our answer will have the same sign as 7 which is negative.

So, $-(7-6)=-1$

$6-7=-1$

Example 4: Evaluate

$-3-4$

The 3 is negative and the 4 is also negative.

Since the signs are the same, we should find the sum of 3 and 4.

Since both values are negative, our answer will also be negative.

So, $-(3+4)=-7$

$-3-4=-7$

Example 5: Evaluate

$1 + (-2)$ **We see parenthesis because we have two operations directly next to each other.

The 1 is positive and the 2 is negative.

Since the signs are opposite, we should find the difference between 1 and 2.

Since the 2 has the larger absolute value, our answer will have the same sign as 2 which is negative.

So, $-(2 - 1) = -1$

$1 + (-2) = -1$

Example 6: Evaluate

$-7 - (-8)$ **We see parenthesis because we have two operations directly next to each other.

Since the two operations directly next to each other are both negatives (minus a negative), we should turn that into a positive and re-evaluate our problem.

$-7 + 8$

The 7 is negative and the 8 is positive. Remember that the sign of a number is the same as the sign the directly precedes the number.

Since the signs are opposite, we should find the difference between 7 and 8.

Since the 8 has the larger absolute value, our answer will have the same sign as 8 which is positive.

So, $+(8 - 7) = +1$

$-7 - (-8) = 1$

Example 8: Evaluate

$4 - (-2)$ **We see parenthesis because we have two operations directly next to each other.

Since the two operations directly next to each other are both negatives (minus a negative), we should turn that into a positive and re-evaluate our problem.

$4 + 2$

The 4 is positive and the 2 is also positive.

Since the signs are the same, we should find the sum of 4 and 2.

Since both values are positive, our answer will also be positive.

So, $+(4+2)=+6$

$4-(-2)=6$

Example 9: Evaluate

$-37+34$

The 37 is negative and the 34 is positive.

Since the signs are opposite, we should find the difference between 37 and 34.

Since the 37 has the larger absolute value, our answer will have the same sign as 37 which is negative.

So, $-(37-34)=-3$

$-37+34=-3$

Example 10: Evaluate

$-45+(-43)$ **We see parentheses because we have two operations directly next to each other.

The 45 is negative and the 43 is also negative.

Since the signs are the same, we should find the sum of 45 and 43.

Since both values are negative, our answer will also be negative.

So, $-(45+43)=-88$

$-45+(-43)=-88$

Example 11: Evaluate

$45 - 19$

The 45 is positive and the 19 is negative. Remember that the sign of a number is the same as the sign the directly precedes the number. A subtraction is the same as a negative number.

Since the signs are opposite, we should find the difference between 45 and 19.

Since the 45 has the larger absolute value, our answer will have the same sign as 45 which is positive.

So, $+(45 - 19) = +26$

$45 - 19 = 26$

Example 12: Evaluate

$-6 - 28$

The 6 is negative and the 28 is also negative.

Since the signs are the same, we should find the sum of 6 and 28.

Since both values are negative, our answer will also be negative.

So, $-(6 + 28) = -34$

$-6 - 28 = -34$

Example 13: Evaluate

$44 + (-22)$

**We see parenthesis because we have two operations directly next to each other.

The 44 is positive and the 22 is negative.

Since the signs are opposite, we should find the difference between 44 and 22.

Since the 44 has the larger absolute value, our answer will have the same sign as 44 which is positive.

So, $+(44 - 22) = +22$

$44 + (-22) = 22$

Example 14: Evaluate

$-5-(-44)$ **We see parenthesis because we have two operations directly next to each other.

Since the two operations directly next to each other are both negatives (minus a negative), we should turn that into a positive and re-evaluate our problem.

$-5+44$

The 5 is negative and the 44 is positive. Remember that the sign of a number is the same as the sign the directly precedes the number.

Since the signs are opposite, we should find the difference between 5 and 44.

Since the 44 has the larger absolute value, our answer will have the same sign as 44 which is positive.

So, $+(44-5)=+39$

$-5-(-44)=39$

Example 15: Evaluate

$13-(-1)$ **We see parenthesis because we have two operations directly next to each other.

Since the two operations directly next to each other are both negatives (minus a negative), we should turn that into a positive and re-evaluate our problem.

$13+1$

The 13 is positive and the 1 is also positive.

Since the signs are the same, we should find the sum of 13 and 1.

Since both values are positive, our answer will also be positive.

So, $+(13+1)=+14$

$13-(-1)=14$

Example 16: Evaluate

$16 - 36$

The 16 is positive and the 36 is negative. Remember that the sign of a number is the same as the sign the directly precedes the number. A subtraction is the same as a negative number.

Since the signs are opposite, we should find the difference between 16 and 36.

Since the 36 has the larger absolute value, our answer will have the same sign as 36 which is negative.

So, $-(36 - 16) = -20$

$16 - 36 = -20$

Adding and Subtracting Rational Numbers Rules:

1. Turn all mixed numbers and integers into improper fractions.
2. Multiply numerators and denominators to obtain a common denominator between the improper fractions.
3. Follow the rules for adding and subtracting integers to find the numerator. The denominator will be the common denominator that you obtained in step #2.
4. Simplify the fraction (make sure the numerator and denominator are not divisible by the same values).
5. If your answer is an improper fraction turn it into a mixed number.

Example 17: Evaluate

$$-\frac{10}{7} + 4\frac{3}{7}$$

1. $-\frac{10}{7} + \frac{31}{7}$ **Remember to turn $4\frac{3}{7}$ into an improper fraction, we use $4 \cdot 7 + 3 = 31$ to find the numerator.
2. We already have a common denominator (7), so we can skip this step.
3. The numerators are -10 and 31.
   The 10 is negative and the 31 is positive.
   Since the signs are opposite, we should find the difference between 10 and 31.
   Since the 31 has the larger absolute value, our answer will have the same sign as 31 which is positive.
   So, $+(31 - 10) = +21$

$$-\frac{10}{7}+\frac{31}{7}=\frac{21}{7}$$

4. 21 and 7 are both divisible by 7.
   $$\frac{21 \div 7}{7 \div 7}=\frac{3}{1}$$
5. $\frac{3}{1}$ is an improper fraction. So, we divide. $3 \div 1 = 3$

$$-\frac{10}{7}+4\frac{3}{7}=3$$

<u>Example 18: Evaluate</u>

$$\frac{3}{5}-\left(-\frac{5}{6}\right)$$

1. We don't have any mixed numbers or integers in this problem, so we can skip this step.
2. The LCM of 5 and 6 is 30. So, we will multiply 6 into the numerator and denominator of $\frac{3}{5}$ and 5 into the numerator and denominator of $-\frac{5}{6}$.
   $$\frac{6\cdot 3}{6\cdot 5}-\left(-\frac{5\cdot 5}{6\cdot 5}\right)$$
   $$\frac{18}{30}-\left(-\frac{25}{30}\right)$$
3. Since the two operations directly next to each other are both negatives (minus a negative), we should turn that into a positive and re-evaluate our problem.
   $$\frac{18}{30}+\frac{25}{30}$$
   The 18 is positive and the 25 is also positive.
   Since the signs are the same, we should find the sum of 18 and 25.
   Since both values are positive, our answer will also be positive.
   So, $+(18+25)=+43$
   $$\frac{18}{30}+\frac{25}{30}=\frac{43}{30}$$
4. 43 and 30 do not have a common divisor, so we can skip this step.
5. $\frac{43}{30}$ is an improper fraction. So, we divide.
   $$\begin{array}{r} 1 \\ 30\overline{)\,43} \\ \underline{-30} \\ 13 \end{array} \qquad\qquad 1\tfrac{13}{30}$$

$$\frac{3}{5}-\left(-\frac{5}{6}\right)=1\frac{13}{30}$$

Example 19: Evaluate

$$-2+\frac{8}{7}$$

1. $-\frac{2}{1}+\frac{8}{7}$ **Remember to turn -2 into an improper fraction, we put the integer over 1.
2. The LCM of 1 and 7 is 7. So, we will multiply 7 into the numerator and denominator of $-\frac{2}{1}$. We do not need to multiply anything into $\frac{8}{7}$ since it's denominator is already 7.
   $-\frac{7\cdot 2}{7\cdot 1}+\frac{8}{7}$
   $-\frac{14}{7}+\frac{8}{7}$
3. The numerators are -14 and 8.
   The 14 is negative and the 8 is positive.
   Since the signs are opposite, we should find the difference between 14 and 8.
   Since the 14 has the larger absolute value, our answer will have the same sign as 14 which is negative.
   So, $-(14-8)=-6$
   $-\frac{14}{7}+\frac{8}{7}=-\frac{6}{7}$
4. 6 and 7 do not have a common divisor, so we can skip this step.
5. $-\frac{6}{7}$ is not an improper fraction, so we can skip this step.

$$-2+\frac{8}{7}=-\frac{6}{7}$$

Example 20: Evaluate

$$\frac{3}{7}+\left(-1\frac{1}{6}\right)$$

1. $\frac{3}{7}+\left(-\frac{7}{6}\right)$ **Remember to turn $1\frac{1}{6}$ into an improper fraction, we use $1\cdot 6+1=7$ to find the numerator.
2. The LCM of 7 and 6 is 42. So, we will multiply 6 into the numerator and denominator of $\frac{3}{7}$ and 7 into the numerator and denominator of $-\frac{7}{6}$.
   $\frac{6\cdot 3}{6\cdot 7}+\left(-\frac{7\cdot 7}{6\cdot 7}\right)$
   $\frac{18}{42}+\left(-\frac{49}{42}\right)$
3. The numerators are 18 and -49.
   The 18 is positive and the 49 is negative.

Since the signs are opposite, we should find the difference between 18 and 49.
Since the 49 has the larger absolute value, our answer will have the same sign as 49 which is negative.
So, $-(49-18)=-31$
$\frac{18}{42}+\left(-\frac{49}{42}\right)=\frac{-31}{42}$

4. 31 and 42 do not have a common divisor, so we can skip this step.
5. $-\frac{31}{42}$ is not an improper fraction, so we can skip this step.

$\frac{3}{7}+\left(-1\frac{1}{6}\right)=-\frac{31}{42}$

Example 21: Evaluate

$\left(-\frac{1}{3}\right)-4\frac{3}{8}$

1. $\left(-\frac{1}{3}\right)-\frac{35}{8}$ **Remember to turn $4\frac{3}{8}$ into an improper fraction, we use $4\cdot 8+3=35$ to find the numerator.
2. The LCM of 3 and 8 is 24. So, we will multiply 8 into the numerator and denominator of $-\frac{1}{3}$ and 3 into the numerator and denominator of $-\frac{35}{8}$.
   $-\frac{8\cdot 1}{8\cdot 3}-\frac{35\cdot 3}{8\cdot 3}$
   $-\frac{8}{24}-\frac{105}{24}$
3. The numerators are -8 and -105
   The 8 is negative and the 105 is also negative.
   Since the signs are the same, we should find the sum of 8 and 105.
   Since both values are negative, our answer will also be negative.
   So, $-(8+105)=-113$
   $-\frac{8}{24}-\frac{105}{24}=-\frac{113}{24}$
4. 113 and 24 do not have a common divisor, so we can skip this step.
5. $-\frac{113}{24}$ is an improper fraction. So, we divide. (Ignore the negative for the algorithm and add it back into your final answer.)

$$\begin{array}{r} 4 \\ 24\overline{)113} \\ -96 \\ \hline 17 \end{array}$$

$4\frac{17}{24}$

$\left(-\frac{1}{3}\right)-4\frac{3}{8}=-4\frac{17}{24}$

Example 22: Evaluate

$$-\frac{1}{2}-\frac{9}{7}$$

1. We don't have any mixed numbers or integers in this problem, so we can skip this step.
2. The LCM of 2 and 7 is 14. So, we will multiply 7 into the numerator and denominator of $-\frac{1}{2}$ and 2 into the numerator and denominator of $-\frac{9}{7}$.
   $$-\frac{7\cdot 1}{7\cdot 2}-\frac{9\cdot 2}{7\cdot 2}$$
   $$-\frac{7}{14}-\frac{18}{14}$$
3. The numerators are -7 and -18.
   The 7 is negative and the 18 is also negative.
   Since the signs are the same, we should find the sum of 18 and 25.
   Since both values are negative, our answer will also be negative.
   So, $-(7+18)=-25$
   $$-\frac{7}{14}-\frac{18}{14}=-\frac{25}{14}$$
4. 25 and 14 do not have a common divisor, so we can skip this step.
5. $-\frac{25}{14}$ is an improper fraction. So, we divide. (Ignore the negative for the algorithm and add it back into your final answer.)
   $$\begin{array}{r} 1 \\ 14\overline{)\,25} \\ \underline{-14} \\ 11 \end{array} \qquad 1\frac{11}{14}$$

$$-\frac{1}{2}-\frac{9}{7}=-1\frac{11}{14}$$