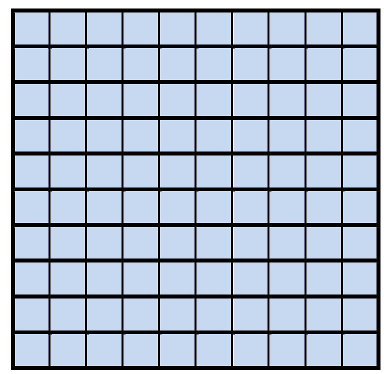
0.3b Class Activity: Multiplying Muli-Digit Decimals

You may choose to do all three parts of this section or just one or two depending on the needs of your students and time constraints in your classroom. In previous grades, students learned to multiply multi-digit numbers with decimal factors limited to hundredths and decimal products limited to thousandths. This lesson will help students extend their previous understanding of decimal multiplication to numbers that have decimals beyond hundredths. Throughout the tasks and exercises in this lesson it is important to not only focus on the procedure of placing the decimal point in the correct position but to accompany the procedure with an understanding of place value, number sense, and operation.

Part 1

Use the model given to discuss the following questions.

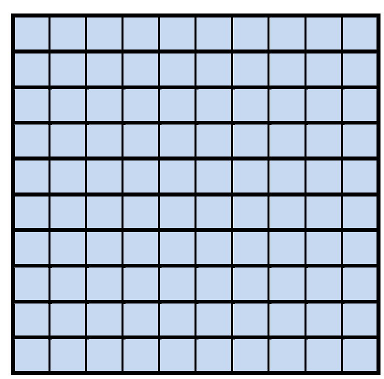
1. Assume the side length of the large square is 10.
   What is the area of the large square? What is the side length of a small square? What is the area of the small square?



Encourage students to label the model with their findings and to justify their reasoning. Possible arguments are given below.

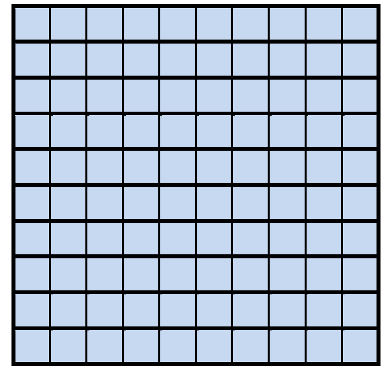
Since the side length of the large square is 10 units long then the area of the large square is $10 \times 10 = 100$. The side length of the large square is 10 units long and there are 10 small squares on each side length; this means that the side length of a small square is $10 \div 10 = 1$. Since the side length of the small square is 1 then the area of the small square is $1 \times 1 = 1$. Students might also argue that the area of the small square is 0.01 or $\frac{1}{100}$ of the area of the large square. $0.01 \times 100 = 1 \text{ or } 100 \div 100 = 1$.

2. Now assume the side length of the large square is 100.
   What is the area of the large square? What is the side length of a small square? What is the area the small square?

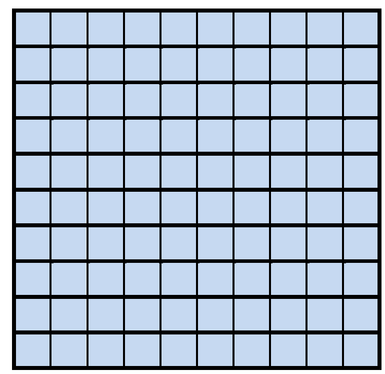


The area of the large square is 10,000 because the side length of the large square is 100 units long and $100 \times 100 = 10{,}000$. The side length of the small square is 10. Since there are 10 small squares on each side length then the side length of a small square is $100 \div 10 = 10$. Since the side length of the small square is 10 then the area of the small square is $10 \times 10 = 100$. Students might also reason that the area of the small square is 0.01 or $\frac{1}{100}$ of the area of the large square. $0.01 \times 10{,}000 = 10 \text{ or } 10{,}000 \div 100 = 10$.

3. Now assume the side length of the large square is 1.
   What is the area of the large square? What is the side length of a small square? What is the area the small square?
   The area of the large square is 1. Since the side length of the large square is 1 unit long then $1 \times 1 = 1$. Since the side length of the large square is 1 unit long and there are 10 small squares on each side length then the side length of a small square is $1 \div 10 = 0.1$ The area of the small square is 0.01, this is because the side length of the small square is 0.1 and $0.1 \times 0.1 = 0.01$. If students do not have a strong understanding of decimal multiplication from previous grades they might reason that the area of the small square is 0.01 or $\frac{1}{100}$ of the area of the large square. $0.01 \times 1 = 0.01$ *or* $1 \div 100 = 0.01$.



4. Assume the side length of the large square is 0.1.
   What is the area of the large square? What is the side length of a small square? What is the area the small square?
   The area of the large square is 0.01 because the side length of the large square is 0.1 units long and $0.1 \times 0.1 = 0.001$. The side length of the small square is 0.01 because the length of the large square is 0.1 units long and there are 10 small squares on each side. $0.1 \div 10 = 0.01$ or 0.1 of 0.1. Since the side length of
   the small square is 0.01 then the area is $0.01 \times 0.01 = 0.0001$. Most students will not know how to multiply 0.01 and 0.01 since they only produced products up to thousandths in previous grades. However, they can reason that the area of the small square is 0.01 or $\frac{1}{100}$ of the area of the large square. $0.01 \times 0.1 = 0.0001$ *or* $0.01 \div 100 = 0.0001$.

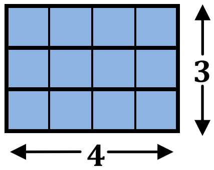


5. Summarize your thinking by stating the following products
   a. $10 \times 10 =$ 100
   b. $1 \times 1 =$ 1
   c. $0.1 \times 0.1 =$ 0.01
   d. $0.01 \times 0.01 =$ 0.0001

At this point students should be able to give an argument for why $0.1 \times 0.1 = 0.01$ and $0.01 \times 0.01 = 0.0001$ from a geometric perspective as related to the base ten grids used above. Also they should be able to tell you why the product is smaller than the two factors. They might also begin reasoning about how the placement of the decimal point in the product is related to the total number of decimal places to the right of zero in the factors. This will be solidified in the next part of this section.

Part 2

1. Use the model below to answer the questions that follow.



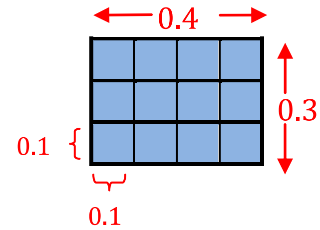
   a. What is the length of each unit?
   Each unit has a length of 1

   b. What is the area of each small square?
   The area of each small square is 1.

   c. What is the area of the rectangle?
   The area of the rectangle is 12.

   d. Write a multiplication equation that represents the dimensions and area of this rectangle.
   $4 \times 3 = 12$

2. Now suppose the length of each unit is 0.01.



   a. Label the length of each unit (small square) on the rectangle and label the side lengths of the rectangle.

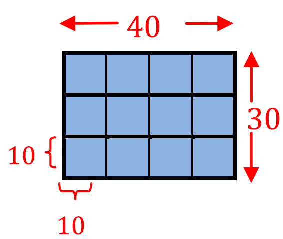
   b. What is the area of each small square?
   The area of each small square is 0.01.

   c. What is the area of the rectangle?
   0.12

   d. Write a multiplication equation that represents the area of the rectangle above.
   $0.4 \times 0.3 = 0.12$

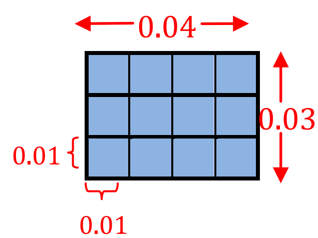
For problems 3 through 8 label each rectangle with the given dimensions so that it represents the multiplication equation. Label the dimensions of each unit (small square). Find the area of each small square and the area of each rectangle that represents the solution to each equation. Then write the solution to the equation.

3. $40 \times 30 =$ 1200



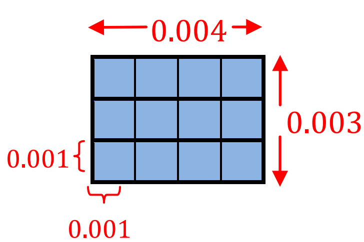
Area of the small square:100
Area of the rectangle:1200

4. $0.04 \times 0.03 =$ 0.0012



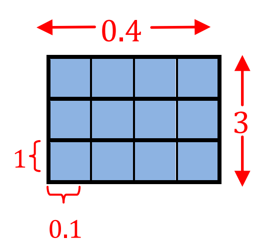
Area of the small square: 0.0001
Area of the rectangle:0.0012

5. $0.003 \times 0.004 =$ 0.000012



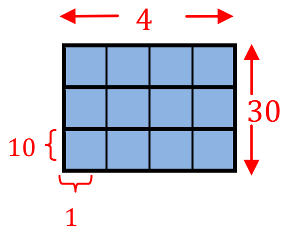
Area of the small square:0.000001
Area of the rectangle:0.000012

6. $0.4 \times 3 =$ 0.12



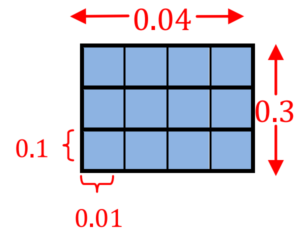
Area of the small square: 0.1
Area of the rectangle:0.12

7. $4 \times 30 = 120$



Area of the small square: 10
Area of the rectangle: 120

8. $0.04 \times 0.3 = 0.012$



Area of the small square: 0.001
Area of the rectangle: 0.012

The purpose of this task is for students to think about the placement of the decimal in the product when multiplying decimals. Students may use a variety of strategies to obtain the products above.

- They may use reasoning used in previous grades about patterns in the number of zeros that occur when multiplying whole numbers by powers of ten to also reason about the placement of the decimal point when a decimal is multiplied by a power of ten.
- They might also arrive at the final product by finding the area of each unit (small square) as learned in Part 1 of this section and then add the area of all the small squares to get the final product.
- Encourage them to check their answer using estimation reasoning as well. For example, if you are multiplying 0.04×0.3 you are taking four hundredths of three tenths therefore your product is going to go be in the thousandths.

A common misconception may arise if students infer that the decimal point is placed in regards to the number of zeros and not the number of places to the right of the decimal point. Also watch out for problems that might occur if students are unclear on the placement of the decimal point, for example some students might state that $0.5 \times 0.02 = 0.001$ rather than the correct product of 0.01 which arises from 0.010.

Be sure to clearly illustrate the connection between the total number of decimals places to the right of 0 in the factors to the number of places to the right of 0 in the product.

For example $\underbrace{0.04 \times 0.3}_{\text{Total of 3 places to the right of the decimal point}} = \underbrace{0.012}_{\text{3 places in the product}}$

Part 3

1. Use the fact that $16 \times 12 = 192$ to find each product. Justify your answer using an estimation argument and by discussing the placement of the decimal point or the expected number of zeros in the product.

Possible estimation arguments are given. It is not expected that students "write" all of their reasoning below. The questions are intended to lead a verbal discussion. You could divide your class into groups and assign each group one product. Have them present their reasoning to the class either verbally or on poster paper.

a. $16 \times 1.2 = 19.2$
1.2 is a little more than 1 so the product of 16 and 1.2 should be more than the product of 16 and 1. 19.2 is a reasonable estimate for this. Or 1.2 is ten times smaller than 12 so our product should be 10 times smaller than 192. This can be obtained by moving the decimal one place to the left. 19.2 is ten times less than 192. In other words, one of the factors has a decimal point that is one place to the left of 12, thus our product should have a decimal that is moved once place to the left.

b. $160 \times 12 = 1920$
Since 160 is ten times as large as 16 then the product should be ten times as large at 192. One of the factors has an additional zero so that product should have an additional zero.

c. $160 \times 120 = 19200$
Since the two factors are ten times as large as the original factors then the product should be 100 times bigger than the original product. In other words, each factor has an additional zero so the product should have 2 additional zeros.

d. $1600 \times 12 = 19200$
Since 1600 is 100 times as big as 16 then our product should be 100 times bigger. In other words there are two additional zeros in the first factor so our product will have two additional zeros. Relate this to the problem above.

e. $1.6 \times 1.2 = 1.92$
1.6 is close to 2 and 1.2 is close to 1. So the product should be to 2. 1.92 is a reasonable estimate for this. 1.6 is ten times smaller than 16 and 1.2 is ten times smaller than 12. Thus our product should be 100 times smaller 192. This can be obtained by moving the decimal point two place to the left of the original product, giving us 1.92. In other words each factor has a decimal point the is 1 moved one place to the left of the original factors. Making a total of two places to the left for the factors, this means our product must have a decimal point that is moved two places to the left.

f. $0.16 \times 1.2 = 0.192$
This product will be less than 1 because we are taking 0.16 of 1.2. Since 0.16 is one hundred times smaller than 16 and 1.2 is ten times smaller than 12. Then our product will be 1000 times smaller than the original product. This can be obtained by moving the decimal point three spaces to the left in the original product yielding 0.192. In other words the first factor has a decimal point that has been moved two places to the left, the second factor has a decimal point that has been moved one place to the left. So we must move the decimal point in the product a total of three places to the left.

g. $0.16 \times 0.12 = 0.0192$
This product will be much less than one since we are taking 0.16 of 0.12. Since 0.16 is one hundred times smaller than 15 and 0.12 is also 100 times smaller than 12. Then our product will be 10,000 times smaller than 192. This can be obtained by moving the decimal in the original product 4 places to the left, yielding 0.0192. In other words the first factors has a decimal point that has been moved two place to the left, the second factor has a decimal point that has been moved two places to the left. So we must move the decimal point the product a total of four places to the left.

2. Dallin has begun to do the following multiplication problem. His work is shown below; he does not know where to place the decimal point in the product. Correctly place the decimal point for him and justify your answer.

$$
\begin{array}{r}
1 \\
\not{1}\,\not{3} \\
4.\underline{37} \\
\times \quad 1.\underline{25} \\
\hline
2185 \\
+ \quad 8740 \\
\hline
10925 \\
+43700 \\
\hline
5.\underline{4}\underline{6}\underline{2}\underline{5}
\end{array}
$$

The decimal must be placed 4 places to the left in the product because the first factor has the decimal placed two places to the left and the second factor has the decimal placed two places to the left, making a total of 4 places to the left. Also if we estimate the product by rounding the products we know that $4 \times 1 = 4$ so our final product should be close to 4. Thus we must place the decimal point between 5 and 4.

Directions: For each problem estimate the product. Then find the product by changing the decimals to fractions. Check your answer by using the multiplication algorithm.

3. 25.62×11.7

Estimation: You can round 25.62 to 30 and 11.7 to 10. This gives us $30 \times 10 = 300$. This product can be estimated by reasoning that $3 \times 1 = 3$ and each factor has one zero so our product will have two zeros, this yields 300.

Convert to Fractions: $25.62 = 25\frac{62}{100}$ and $11.7 = 11\frac{7}{10}$. Thus $25\frac{62}{100} \times 11\frac{7}{10} = \frac{2562}{100} \times \frac{117}{10} = \frac{299754}{1000}$. Since the denominator is 1000 the last digit should be in the thousandths place when writing the decimal. Thus $\frac{299754}{1000} = 299.754$

Standard Algorithm: The placement of the decimal point can be obtained several ways. The estimation above yielded a product close to 300, which would place the decimal between the 9 and 7. Or, since 25.63 is 100 times smaller than 2562 and 11.7 is ten times smaller than 117 than our final product must be $100 \times 10 = 1000$ times smaller than 299,754. We can do this by moving the decimal point to the left three spaces. They could also reason that there are 2 decimal digits in the first factor and 1 decimal digit in the second factor; this will make 3 decimal digits in the product.

```
  3 4 1
   25.62
×   11.7
--------
   17934
   25620
+ 256200
--------
 299.754
```

4. You work part time at a book store and get paid $12.05 per hour. In the entire month of March you worked 78.25 hours. How much money did you make in March?

Estimation: You can round 12.05 to 12 and 78.25 to 80. This gives us $12 \times 80 = 960$. You can estimate this product by reasoning that $12 \times 8 = 96$ and the second factor has one zero so our product will have one zero. This yields about $960 made in March.

Convert to Fractions: $12.05 = 12\frac{5}{100}$ and $78.25 = 78\frac{25}{100}$. Thus $12\frac{5}{100} \times 78\frac{25}{100} = \frac{1205}{100} \times \frac{7825}{100} = \frac{9429125}{10000}$. The denominator is 10,000 so the last digit should be in the ten thousandths place when writing the decimal. Thus $\frac{9429125}{10000} = 942.9125$

Standard Algorithm: The placement of the decimal point can be obtained several ways. The estimation above yielded a product close to 300, which would place the decimal between the 9 and 7. Or, since 25.63 is 100 times smaller than 2,562 and 11.7 is ten times smaller than 117 than our final product must be $100 \times 10 = 1000$ times smaller than 299754. We can do this by moving the decimal point to the left three spaces. They could also reason that there are 2 decimal digits in the first factor and 1 decimal digit in the second factor; this will make 3 decimal digits in the product.

```
 1  3
 1  4
    1
 1  2
   12.05
   78.25
--------
    6025
   24100
  964000
 8435000
--------
942.9125
```