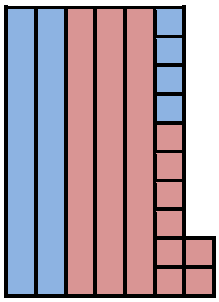
0.3a Class Activity: Adding and Subtracting Multi-Digit Decimals

As students use models to better understand the addition algorithm they use repeated reasoning to understand that as you add numbers with decimals beyond hundredths the same principles of place value hold true. When they connect the models to the algorithm they structure their numbers by lining them up appropriately to make use of place value.

Marta has created the model below. She claims that this model can be used to represent the sum of 24 and 38.

You may need to point out that the first addend (24) is shaded blue and the second addend (38) is shaded pink. The sum is shown by combining all the rods and squares together. If needed, you can use base-ten blocks to model this as well.



In previous grades students used base-ten blocks to model addition for whole numbers and numbers with decimals up to hundredths. The blocks and model help students to understand why we "carry" as we bundle into groups of ten when using the addition algorithm. The purpose of this task is to extend that understanding to adding decimals greater than hundredths. Similarly, we can use models to carry and bundle into groups of ten and relate this to the addition algorithm with decimals.

4. If Marta's claim is true, what is the value of the small square?
   A small square has a value of 1 unit.

5. What is the value of a rod (long rectangle)?
   A rod has a value of 10

6. Find the sum of 24 and 38 using the addition algorithm and discuss how this relates to the model above.

$$\begin{array}{r} {\scriptstyle 1} \\ 24 \\ +38 \\ \hline 62 \end{array}$$

Review with your class how the algorithm relates to the model and place value. i.e. That "carrying" the one represents the grouping of the 10 ones into a rod of 10, which is then combined with the other rods of 10.

Using the same model now suppose that the small square represents $\mathbf{0.1}$.

7. What would the value of the rod be?
   The rod would equal 1

8. What would the sum be equal to?
   The sum would equal 6.2

This requires students to really make sense of the manipulative has they change their thinking and they use blocks to represent decimals or parts less than 1.

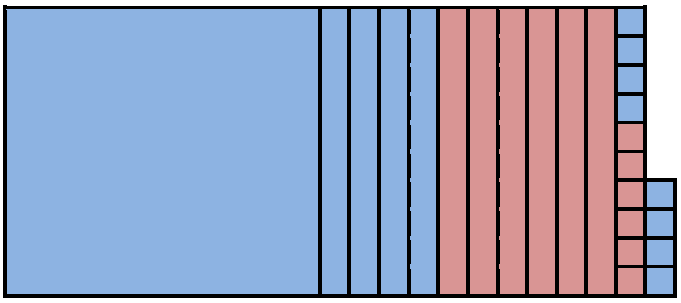
9. Find the sum supposing that the small square represents 0.1 using the addition algorithm and discuss how this relates to the model.

$$\begin{array}{r} {\scriptstyle 1} \\ 2.4 \\ +3.8 \\ \hline 6.2 \end{array}$$

The model shows that the algorithm still holds true for decimals of tenths.

10. What other sums might be represented with this model? Find at least two and for each sum identify what a square represents and what a rod represents.
    Sample answer: $\mathbf{0.24 + 0.38}$, for this sum the value of a square is $\mathbf{0.01}$ and the value of a rod would be $\mathbf{0.1}$.
    Sample answer: $\mathbf{240 + 380}$, for this sum the value of a square is $\mathbf{10}$ and the value of a rod is $\mathbf{100}$.
    Other sums might be $\mathbf{2400 + 380}$, $\mathbf{0.024 + 0.038}$, or even $\mathbf{48 + 76}$. For the last sum the value of small square would be $\mathbf{2}$ and the value of a rod would be $\mathbf{20}$, however, for our purposes we are only going to investigate units with a base of 10.

Study the new model below.



11. Irina claims that the model represents the sum of $\mathbf{1480}$ and $\mathbf{660}$. For this to be true what is the value of a small square, a rod, and large square?
    The small square equals 10
    The rod equals 100
    The large square equals 1000

12. What does the sum equal altogether? Use the addition algorithm to find the sum as well.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 1480 \\ +\ 660 \\ \hline 2140 \end{array}$$

13. Carly states that when she looks at the model she sees the sum of $0.\mathbf{148} + \mathbf{0.066}$. For Carly's statement to be true what is the value of a small square, a rod, and a large square?
    The value of a small square is 0.001
    The value of a rod is 0.01
    The value of a large square is 0.1

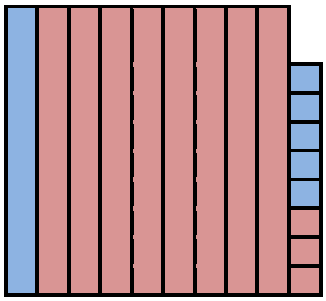
14. What does Carly's sum equal altogether? Use the addition algorithm to find the sum as well. Discuss how the algorithm relates to the model.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 0.148 \\ +0.066 \\ \hline 0.214 \end{array}$$

The model shows that the algorithm still holds true for decimals of thousandths as well. Students should begin to understand that the addition algorithm works for all decimals because we "regroup" bundles of ten has we "carry" numbers in the algorithm.

15. If needed draw a model to find the following sums. Name what the small squares represent and then find each sum using the addition algorithm. You may want to provide graph paper for students to draw their models.

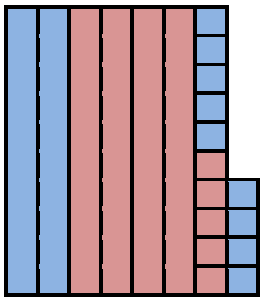
a. $\mathbf{0.015 + 0.083 = 0.098}$



$$\begin{array}{r} 0.015 \\ +0.083 \\ \hline 0.098 \end{array}$$

The small square = 0.001

b. $\mathbf{0.029 + 0.045 = 0.074}$

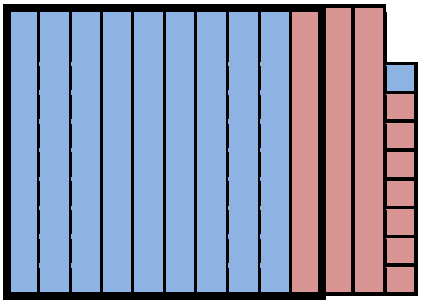


The small square = 0.001

c. $\mathbf{0.37 + 0.048 = 0.419}$

The small square = 0.001

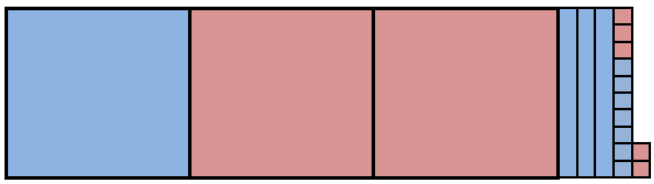
d. $\mathbf{0.091 + 0.037 = 0.128}$



$$\begin{array}{r} {\scriptstyle 1} \\ 0.091 \\ +0.037 \\ \hline 0.128 \end{array}$$

The small square = 0.001

e. $\mathbf{1.37 + 2.05} = 3.42$



The small square = 0.01

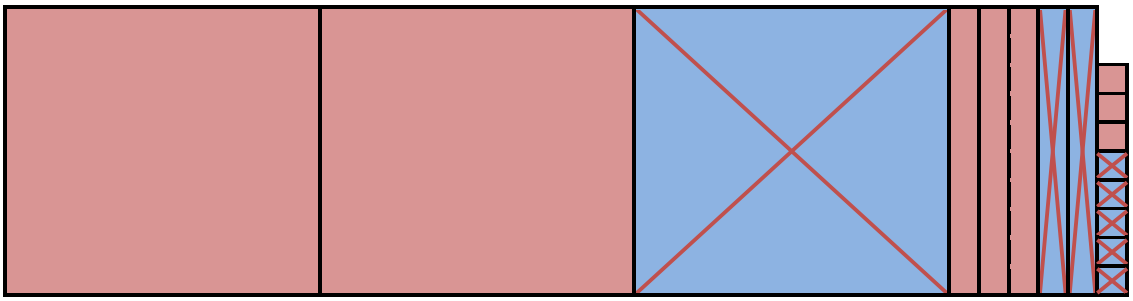
f. $\mathbf{0.103 + 0.0091 = 0.1221}$

The model for this sum would require either really big 10 x 10 squares or really small squares that represent 0.0001. Encourage students to use the algorithm.

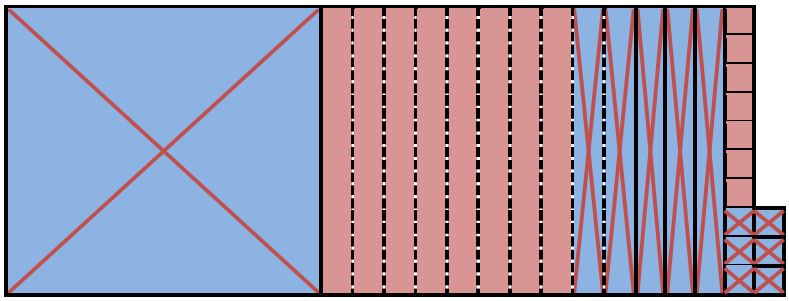
13. Draw models to find the following differences. Then find each difference using the subtraction algorithm.

The purpose of the problems below is for students to see that the same principles hold true for the subtraction algorithm as well. Place value allows us to regroup and unbundle in order to "borrow". This will work for any decimals. The entire model is the minuend and the subtrahend is shown in blue. The final difference is what is leftover (shown in pink)

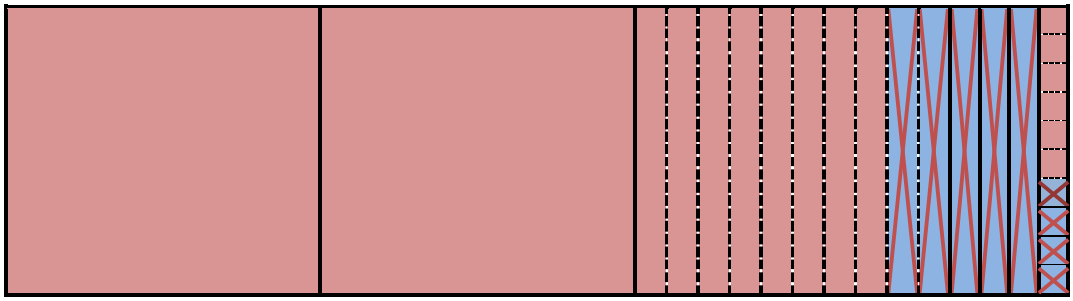
a. $\mathbf{0.358 - 0.125 = 0.233}$



b. $\mathbf{0.243 - 0.156 = 0.087}$



c. $\mathbf{0.34 - 0.054 = 0.286}$



14. Bev has $35.65. She buys a candy bar for $0.89 and package of pens for $3.56.

    a. How much money did Bev spend?
    Bev spent $4.45

    b. How much money does she have leftover?
    Bev has $31.20

15. Peyton is driving to college, her college is $345\frac{11}{50}$ miles away from her parent's home. She has already driven $24\frac{9}{10}$ miles. How much further does she need to drive? Estimate your answer before calculating.
    $345\frac{11}{50} - 24\frac{9}{10} = 345.21 - 24.9 = 10.31$

This problem is a good example of how changing mixed fraction into decimals is sometimes easier than keeping the numbers as fractions and finding a common denominator. Talk about how denominators of $2, 4, 5, 10, 20, 25, 50, 100, 1000, 10{,}000$ lend themselves to easy decimal conversion. Be sure to solidify the addition and subtraction algorithm in the problems above. Students will have the opportunity to practice fluency in adding and subtracting decimals in their homework assignment for this section.