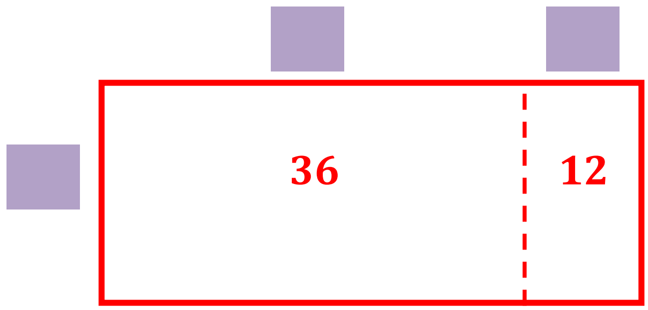
0.2e Class Activity: Using the Distributive Property To Find Equivalent Expressions

In this lesson students begin to make a connection between finding missing numbers in a model to the process of "factoring". In other words; to find the missing numbers they must find a common factor between the two addends in the sum. You can write an equivalent expression to the sum/area as the product of this common factor and a sum of two addends. They will eventually learn that the desired equivalent expression is the one with the GCF as one of the factors, as it is the most useful in the future when rewriting or simplifying expressions or fractions with polynomials.

1. Use the distributive property to write all the equivalent expressions for the sum of $(36 + 12)$. If needed draw a model to reference. Encourage students to try and find the equivalent expressions by just looking at the sum and not using the model. However, the model may be needed to reference so that students clearly understand what terms you are discussing.



Discuss how you can find the equivalent expression without the model. The statements below can aide you in your discussion.

- If you are given just the sum of $36 + 12$ without the model how would you determine what number represents the width on the rectangle (the number outside the parentheses)? This number must be common factor to 36 and 12.
- Use divisibility rules to quickly determine common factors. Start with a common factor of 2. How do you know what addends represent the length of the rectangle (the numbers in the sum inside the parentheses)? These addends are the other factors that multiply with the factor of 2 to get a product of 36 and 12. What number times 2 is 36? What number times 2 is 12? Our addends are 18 and 6. Thus $(36 + 12) = 2(18 + 6) = 50$.
- Move onto the common factor of 3 and use the same reasoning to come up with another equivalent expression. $(36 + 12) = 3(12 + 4) = 50$.
- Repeat the same reasoning with all common factors of 36 and 12.
  $1(36 + 12) = 36 + 12 = 50$
  $2(18 + 6) = 36 + 12 = 50$
  $3(12 + 4) = 36 + 12 = 50$
  $4(9 + 3) = 36 + 12 = 50$
  $6(6 + 2) = 36 + 12 = 50$
  $12(3 + 1) = 36 + 12 = 50$

©2017 University of Utah Middle School Math Project in partnership with the Utah State Office of Education. Licensed under Creative Commons, cc-by.

2. Use the distributive property to find all the equivalent expressions for $24 + 32$
Once again encourage students to use divisibility rules to find common factors of 24 and 32
   $1(24 + 32) = 24 + 32 = 56$
   $2(12 + 16) = 24 + 32 = 56$
   $4(6 + 8) = 24 + 32 = 56$
   $8(3 + 4) = 24 + 32 = 56$

3. Examine the equivalent expressions for the sum in number 2 above. Circle the expression that contains a factor that is the GCF of 24 and 32? What is the other factor in this product? How does this factor partner differ from the other factor partners in the other equivalent expressions?
Talk about how the expression $8(3 + 4)$ is the only product that contains the GCF of 24 and 32. In this expression the factor partner to 8 is $(3 + 4)$. This is the only factor partner that has a sum that contains two addends with no common factor.

4. Examine the equivalent expressions for the sum in number 1 as well. Which expression contains a factor that is the GCF of 36 and 12? How does its factor partner differ from the other factor partners in the other equivalent expressions?
$12(3 + 1) = 36 + 12 = 50$ The same is true; the expression that contains a factor that is the GCF has a factor partner that has a sum that contains two addends with no common factor.

5. Use the distributive property to find all the equivalent expressions for each sum given. Circle the expression that contains a factor that is the GCF of the two addends in the original sum. Check and see if this expression follows the same principle as the expressions with the GCF from numbers 1 and 2 above.

a. $45 + 60$	b. $42 + 70$	c. $20 + 60$
$1(45 + 60) = 45 + 60 = 105$	$1(42 + 70) = 42 + 70 = 132$	$1(20 + 60) = 20 + 60 = 80$
$3(15 + 20) = 45 + 60 = 105$	$2(21 + 35) = 45 + 60 = 105$	$2(10 + 6) = 20 + 60 = 80$
$5(9 + 12) = 45 + 60 = 105$	$7(6 + 10) = 45 + 60 = 105$	$4(5 + 15) = 20 + 60 = 80$
$15(3 + 4) = 45 + 60 = 105$	$14(3 + 5) = 45 + 60 = 105$	$5(4 + 12) = 20 + 60 = 80$
		$10(2 + 6) = 20 + 60 = 80$
		$20(1 + 3) = 20 + 60 = 80$

All of the expressions that contain the GCF have a factor partner that is a sum of two numbers with no common factor.

Discuss with students that finding the equivalent expression that contains the GCF is often the most desirable expression because it will help them to rewrite or simplify expressions that contain variables (monomials or polynomials) or fractions in the future.

If you feel your students need more practice you can ask students to go back through the previous homework assignment and circle all the expressions that contain the GCF for each problem.

©2017 University of Utah Middle School Math Project in partnership with the Utah State Office of Education. Licensed under Creative Commons, cc-by.

6. Find the GCF of the two numbers in each given sum. Use the distributive property to write an equivalent expression to the sum that contains the GCF as one of its factors. How do you know that you found the correct equivalent expression?

a. $42 + 14$ $14(3 + 2)$	b. $36 + 27$ $9(4 + 3)$
c. $55 + 44$ $11(5 + 4)$	d. $16 + 72$ $8(2 + 9)$

You know that you have found an expression that contains the GCF if its factor partner is a sum that contains two numbers that have no common factor.

Factoring out the GCF from a given sum can be viewed from the perspective of multiples as well. If you use the distributive property to write an equivalent expression for a sum by finding the GCF of the two original addends then these two original addends are multiples of the GCF and their sum is also a multiple of its factor partner. We know that the factor partner is a sum of two whole numbers with no common factor. Thus the original sum can be expressed as a multiple of a sum to two whole numbers with no common factor.

For example $36 + 12$ can be written as the equivalent expression of $12(3 + 1)$ by factoring out the GCF of 12. Since 12 is a factor of 36 and 12 then 36 and 12 are both multiples of 12. Also the sum of 36 and 12 is a multiple of the factor partner $(3 + 1)$. The distributive property shows that when you add these two numbers, that are both multiples of 12, their sum (50) is also a multiple of 12. $36 + 12 = 12(3 + 1) = 12(4) = 50$. The following task will help students make this connection.

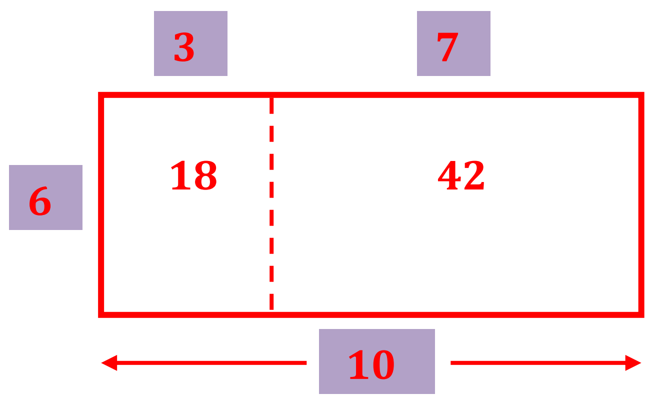
7. Nina was finding multiples of 6. She states,

"18 and 42 are both multiples of 6, and when I add them, I also get a multiple of 6."

$$18 + 42 = 60$$

Explain to Nina why adding two multiples of 6 will always result in another multiple of 6.

18 and 42 are both multiples of 6. Which means that 6 is a factors of both numbers. If you add both numbers you can use the distribute property to show that their sum is also a multiple of 6.
$18 + 42 = 6(3 + 7) = 6 \cdot (10)$



If you have any two multiples of 6 you can use the distributive property to show that the sum of these two numbers is also a multiple of 6. To express this with symbols you can write any two multiples of 6 as

$$6 \cdot a = 6a$$
$$6 \cdot b = 6b$$

where a and b are any whole number. If you add them you get $6a + 6b$. Using the distributive property you can rewrite this expression as $6a + 6b = 6(a + b) = 6 \cdot (a + b)$. This shows that the sum of any two multiples of 6 is always a multiple of 6.

[*This is an Illustrative Mathematics Task](#)

©2017 University of Utah Middle School Math Project in partnership with the Utah State Office of Education. Licensed under Creative Commons, cc-by.