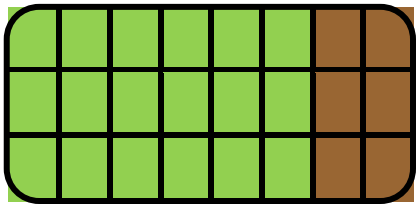
0.2d Class Activity: The Distributive Property

Structure plays a very important role as students look to find equivalent expressions using the distributive property. They must view one or more parts of an expression as a single entity or as a sum or product.
Gordon and Cynthia have made a pan of brownies and have cut them into squares. They have also decided to frost the brownies with mint frosting but have left some of the pieces unfrosted for their friends that don't like frosting as shown.



The purpose of this task is for students to review the distributive property and how it relates to an area model as learned in 3rd grade (3.MD.7C).

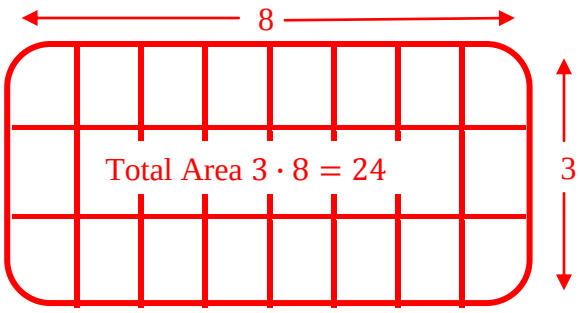
1. How many total brownies are there? Write down a mathematical sentence to show how you arrived at your answer.
   Student answers will vary.
   $3 \times 8 = 24$ This corresponds to the number of rows times the number of columns (Gordon's method)
   $8 + 8 + 8 = 24$ Each row has eight brownies; there are three rows of brownies.
   $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24$ Each column has 3 brownies; there are 8 columns of brownies
   $3 \times 6 + 3 \times 2 = 18 + 6 = 24$ The number of frosted brownies plus the number of unfrosted brownies. (Cynthia's Method)
2. Gordon states that to find the total number of brownies in the pan he counted the number of rows of brownies and multiplied that number by the number of columns of brownies. Write down a mathematical sentence that represents Gordon's thinking.
   $3 \times 8 = 24$

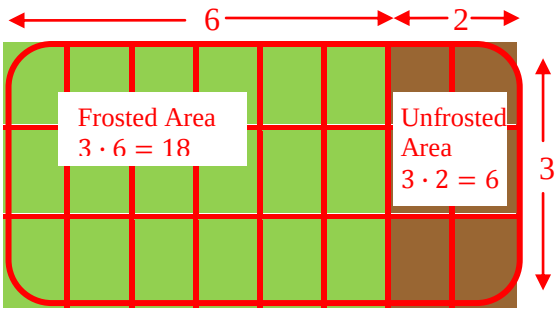
3. Cynthia states that she found the number of frosted brownies first and then she found the number of unfrosted brownies. Once she found the number of each she added them together to find the total. Write down a mathematical sentence that represents Cynthia's thinking.
   $3 \cdot 6 + 3 \cdot 2 = 18 + 6 = 24$

4. Explain why Cynthia and Gordon each arrived at the same answer.

Discuss the connection between the following equations and how they relate to the area model and the distributive property.

$3 \cdot 8 = 24$ $\quad 3(6 + 2) = 3 \cdot 8 = 24$ $\quad 3(6 + 2) = 3 \cdot 6 + 3 \cdot 2 = 18 + 6 = 24$





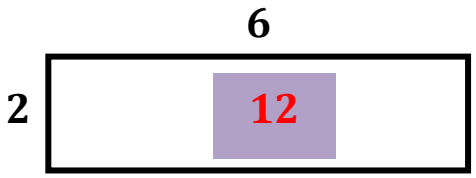
Total Area: $18 + 6 = 24$

Cynthia and Gordon arrived at the same answer because of the Distributive Property. Cynthia's mathematical sentence is equal to Gordon's. $3 \cdot 6 + 3 \cdot 2 = 3(6 + 2) = 3(8) = 24$

Name the factor, product, or addend that is missing from the area model. Then write a mathematical equation that shows the multiplication that the area model represents. Models are not drawn to scale.

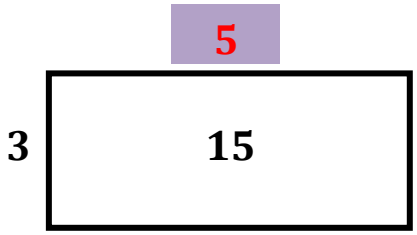
Use the models to review your student's understanding of the distributive property. Finding the missing factors and addends will prepare your students for the next lesson where they will use the distributive property to express a sum of two whole numbers with a common factor as a multiple of a sum of two whole numbers with no common factor. (6.NS.4)

5.



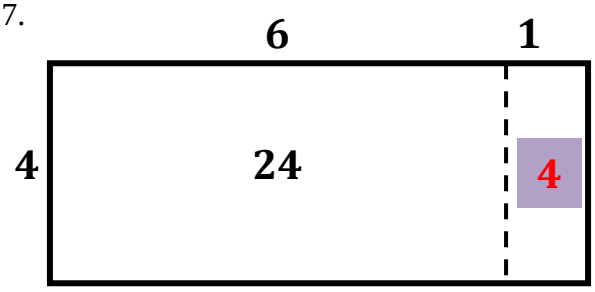
$2 \cdot 6 = 12$

6.



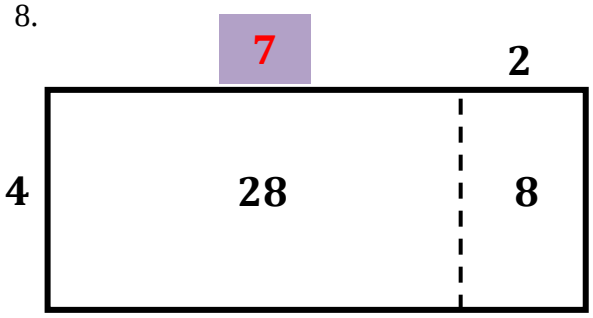
$3 \cdot 5 = 15$

7.



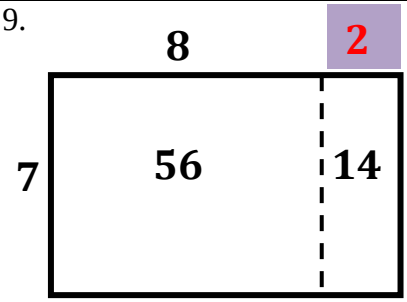
$4 \cdot 6 + 4 \cdot 1 = 24 + 4 = 28$
or $4(6 + 1) = 4(7) = 28$

8.



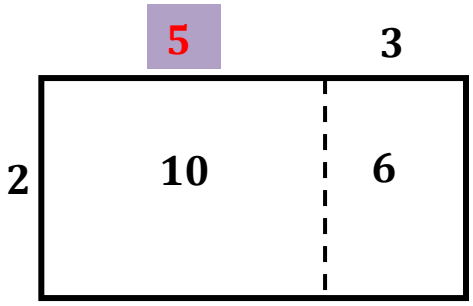
$4 \cdot 7 + 4 \cdot 2 = 28 + 8 = 36$
or $4(7 + 2) = 4(9) = 36$

9.



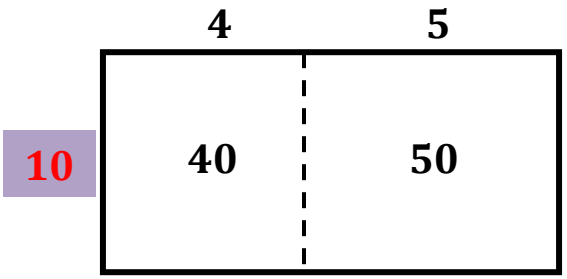
$7 \cdot 8 + 7 \cdot 2 = 56 + 14 = 70$
or $7(8 + 2) = 7(10) = 70$

10.



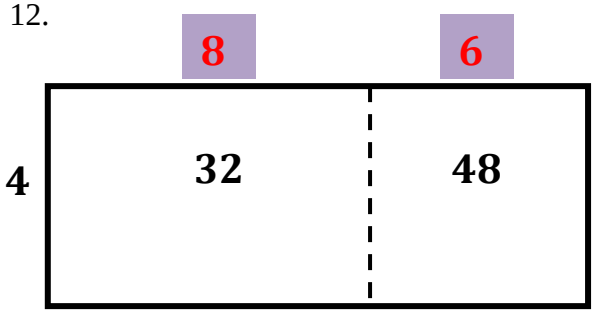
$2 \cdot 5 + 2 \cdot 3 = 10 + 6 = 16$
or $2(5 + 3) = 2(8) = 16$

11.



$10 \cdot 4 + 10 \cdot 5 = 40 + 50 = 90$
or $10(4 + 5) = 10(9) = 90$

12.



$4 \cdot 8 + 4 \cdot 6 = 32 + 24 = 56$
or $4(8 + 6) = 4(14) = 56$

13. Show how the Distributive Property works using the expressions given below.

a. $4(5+7) = 4(5) + 4(7) = 20 + 28 = 48$

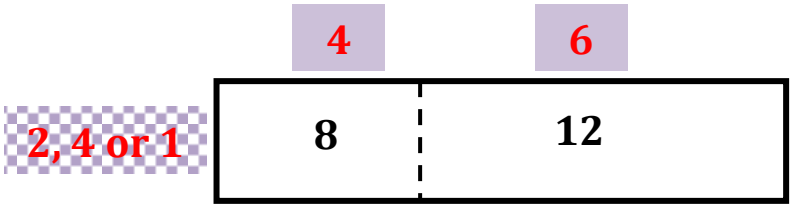
b. $9(7-2) = 9(7) + 9(-2) = 63 + (-18) = 45$

The Distributive Property
To multiply a number by a sum or difference, multiply each number in the sum or difference by the number outside the parentheses.

$$a(b+c) = ab + ac$$
$$a(b-c) = ab - ac$$

For the next few problems students must focus on finding common factors of the two numbers in the given sum. It is okay if students do not immediately identify the greatest common factors at this point. You will notice that there are no problems with subtraction. In 6th grade students are only required to factor a sum of two whole numbers.

14. Lou states that for the area model below the number behind the checkered box is 2.



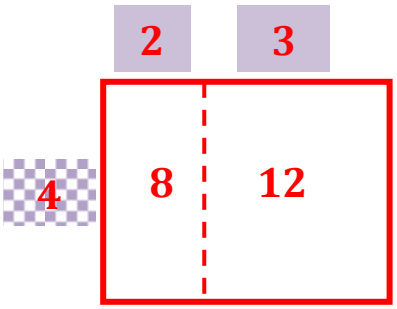
a. Is Lou correct? If so, what must the other missing numbers be? Write a mathematical sentence that describes this area model.
The number behind the checkered box can be 2. If it is, then the number above the 8 must be 4 and the number above the 12 must be 6.
$2(4+6) = 8 + 12 = 20$

b. Are there any other numbers that could be behind the checkered box? If so, what would the other missing numbers be? Write mathematical sentences to describe the area model with these other dimensions as well.
The number behind the checkered box could also be 4. If it is then the number above the 8 must be 2 and the number above the 12 must be 3.
$4(2+3) = 8 + 12 = 20$

The number behind the checkered box could also be 1. If it is then the number above the 8 must be 8 and the number above the 12 must be 12. Be sure to discuss this possibility.
$1(8+12) = 8 + 12 = 20$

c. How do these other numbers affect the length and width of the rectangle? Would the area change? Why or why not?

If the number behind the checkered box is 4 the width of the rectangle would increase (double to be precise) and the length would decrease. The area would stay the same as shown in the model.

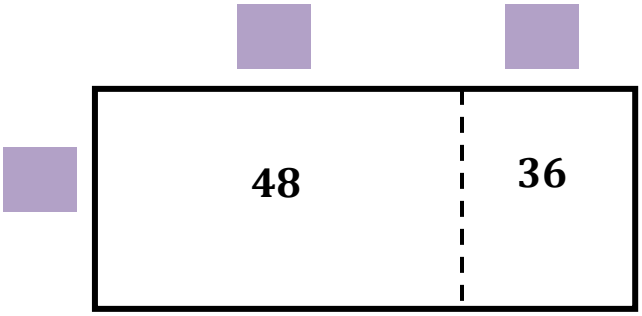


If the number behind the checkered box is 1 the width of the rectangle would decrease and the length would increase. The area would stay the same as shown.

Point out that the area does not change because all of our mathematical expressions are equivalent, in this example they all equal 20.

$$2(4+6) = 4(2+3) = 1(8+12) = 8+12 = 20$$

13. Find the missing numbers for the area problem below. List all possible combinations and write a mathematical sentence for each combination.



$1(48+36) = 48+36 = 84$
$2(24+18) = 48+36 = 84$
$3(16+12) = 48+36 = 84$
$4(12+9) = 48+36 = 84$
$6(8+6) = 48+36 = 84$
$12(4+3) = 48+36 = 84$

At this point it is okay if students are still experimenting or using "guess and check" to find the missing factors and factor addend. They should begin to recognize that they are looking for common factors between the two addends in the sum.