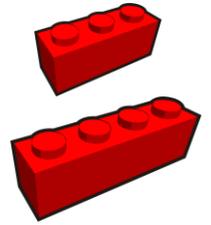
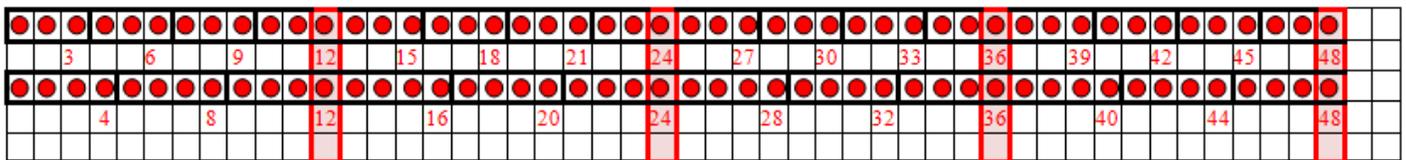


0.2c Class Activity: Least Common Multiple

- Brooks is making a house out of LEGOS. He snaps two rows of LEGOS down onto his mat. In one row he only uses LEGOS with 3 studs; in another row he only uses LEGOS with 4 studs. His mat is only 50 studs long and he wants to make the rows the same length.
 - How many of each type of LEGO, 4 stud and 3 stud, can he use in each row, remembering that the rows need to be the same length? If needed use the grid paper below to help you answer.



As students work through this task review what multiples are as learned in 4th grade. (4.OA.4) Students must recognize that they must identify the multiples of 3 and 4. The grid paper will be helpful for students if they do not recognize this immediately. They might begin by drawing each row of LEGOS and mark off where the rows are equal as shown below. Relate this to multiples of 3 and 4 and then to common multiples of 3 and 4.



Other students might immediately reason that the 3 stud row can have the total stud length be any multiple of 3 that is less than 50: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51
Likewise the 4 stud row can have a total stud length be any multiple of 4 that is less than 50: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52

In order for the rows to be the same length he must makes rows that have the number of studs be common multiples of 3 and 4.

Total number of studs per row (common multiples of 3 and 4)	12	24	36	48
Number of 3 studs block per row	4	8	12	16
Number of 4 stud blocks per row	3	6	9	12

- What is the smallest possible stud length for the rows? How many of each block will be in these rows.

The smallest possible stud length will be the least common multiple of 3 and 4, this is 12. The 3 stud row will have 4 blocks and the 3 stud block will have 3 blocks. Discuss what Least Common Multiple (LCM) is as you answer this question.

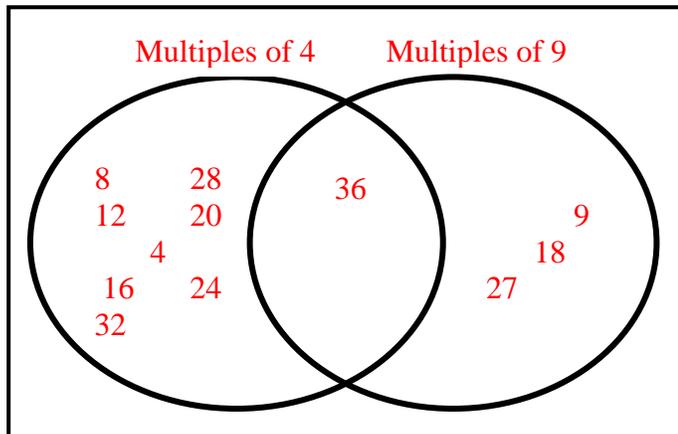
- In the box write down what a Least Common Multiple is.

Least Common Multiple (LCM):

The Least Common Multiple between a pair of numbers is the smallest multiple that the two numbers have in common.

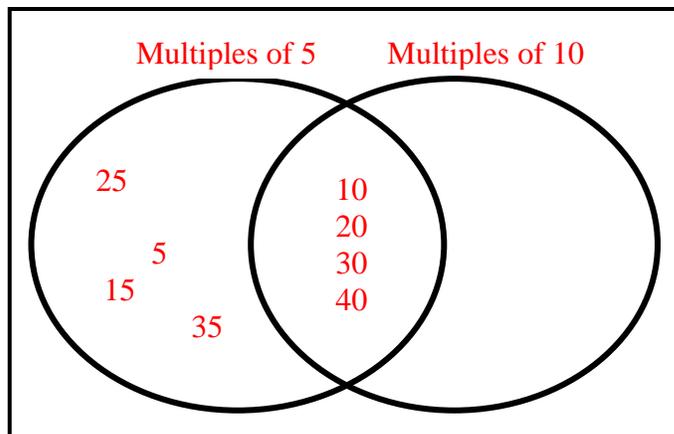
3. Use a Venn diagram to find the LCM of pair of numbers.

a. 4 and 9



The LCM of 4 and 9 is 36

b. 5 and 10



The LCM of 5 and 10 is 10

4. How are factors related to multiples?

A great discussion can surround this question. It is important for students to understand that factors and multiples share a dual relationship. You can use an example to illustrate this; if you list the multiples of 3 you will note that 3 is a factor of all of its multiples.

Multiples of 3: 3, 6, 9, 12, 15, 18, ...

Factors of 3: ③, 1

Factors of 6: 1, 2, ③, 6

Factors of 9: 1, ③, 9

Factors of 12: 1, 2, ③, 4, 6, 12

Factors of 15: 1, ③, 5, 15

Factors of 18: 1, 2, ③, 6, 9, 18

5. Make a list of multiples to find the LCM of each pair of numbers.

a. 9 and 12
36

b. 8 and 6
24

c. 12 and 3
12

d. 4 and 10
20

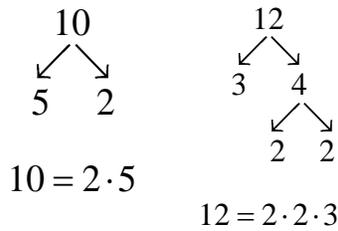
e. 6 and 11
66

f. 9 and 10
90

Sometimes it can be time consuming to list all of the multiples of a number. Rather than writing out a list of multiples for each number you can use the each number's prime factorization to find the least common multiple.

Example

 To find the LCM of 10 and 12 we begin by writing each number as its prime factorization.



Recall we are looking for a multiple of both 10 and 12. That means that 10 and 12 must both be factors of this number we are looking for. Thus this number's prime factorization must include the prime numbers that are in the prime factorizations of 10 and 12.

$$10 = 2 \cdot 5$$

$$12 = 2 \cdot 2 \cdot 3$$

To be a multiple of 10 its prime factorization must include a 2 and a 5 and to be a multiple of 12 its prime factorization must include 2, 2, and 3. In order to be a multiple common to both 10 and 12 we must meet the factor criteria for both numbers by multiplying these factors together. However we want the least common multiple. That means that we only need to include the 2 two times.



In other words, the smallest number that meets both of these conditions is $2 \cdot 2 \cdot 3 \cdot 5 = 60$. Students might wonder why we don't include the other two. We don't need to because we have accounted for all the twos needed and we are looking for the "least" common multiple.

6. Find the prime factorization for each number in a given pair. Then use the prime factorization to find the LCM.

<p>a. 9 and 12</p> <p>$12 = 2 \cdot 2 \cdot 3$ $9 = 3 \cdot 3$</p> <p>A multiple of 12 must include factors of $2 \cdot 2 \cdot 3$ and a multiple of 9 must include factors of $3 \cdot 3$. The smallest multiple that meets both criteria is</p> <p style="text-align: center;">$LCM = 2 \cdot 2 \cdot 3 \cdot 3 = 36$</p>	<p>b. 8 and 18</p> <p>$18 = 2 \cdot 3 \cdot 3$ $8 = 2 \cdot 2 \cdot 2$</p> <p>A multiple of 18 must include factors of $2 \cdot 3 \cdot 3$ and a multiple of 8 must include factors of $2 \cdot 2 \cdot 2$. The smallest multiple that meets both criteria is</p> <p style="text-align: center;">$LCM = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 72$</p>
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<p>c. 20 and 50</p> <p><i>LCM = 100</i></p>	<p>d. 12 and 20</p> <p><i>LCM = 60</i></p>
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7. Two small gears are aligned by a mark drawn down the center of one gear to the center of the other gear. The first gear has 6 teeth and the second gear has 20 teeth. How many revolutions does the first gear need to make so the center lines match up again?
The first gear will need to make 60 rotations before the center lines match up again.

8. Plastic forks come in packages of 10 and plastic knives come in packages of 8. If you want one fork for each knife at a party with none leftover what is the least amount of forks and knives that you need to buy? How many packages of forks and knives will you buy?
All together you will need to buy 40 forks and 40 knives. This will be 4 packages of forks and 5 packages of knives.

