0.2a Class Activity: Divisibility Rules

Understanding how and why divisibility rules work requires students to work with number theory as they look for patterns in place value. They must de-contextualize the properties of a given number and apply it all numbers in general. As they do this they construct arguments and make claims as to why certain properties work for any given number.

1. A marching band has 72 members that will march during halftime in a football game. They need to march in rows with the same number of students in each row. How many ways different ways can the band members be arranged? Explain the arrangements.
   The members can be arranged 6 ways; $1 \times 72,\ 2 \times 36,\ 3 \times 24,\ 4 \times 18,\ 6 \times 12,\ 8 \times 9$. Students might aruge that there are 12 arrangements if they account for changing the direction of the row. For example they could have 2 rows with 36 members in each row or 36 rows with 2 members in each row.

Students must find factors of 72 or numbers that "go into" 72 "evenly". Use this problem to review with your students the terms *factor*, *product*, and *divisible by*.

> Two or more numbers that are multiplied to form a **product** are called **factors**.
>
> $8 \times 9 = 72$ ⟵ Product
>
> ↖ ↗
>
> Factor
>
> This shows that 8 and 9 are factors of 72 because they each divide 72, with no remainder leftover. We say that 72 is **divisible** by 8 and 9.

Often you can test if one number is divisible by another number mentally rather than doing long division to see if there is a remainder. To test a number mentally you can use a Divisibility Rule.

You can help students learn these divisibility rules by using the questions below to lead a class discussion Students can use the space provided to show their thinking throughout the discussion. Very detailed arguments are given as to how and why each divisibility rule works. Refer to pages 12 and 13 in the Mathematical Foundation for further explanation. The extent of these explanations and arguments should be based off of the needs of your students. Before you begin it may be helpful to review with your students that the sum of two even numbers is always even and the sum of two odd numbers is always odd.

Divisibility Rule for 2

- Consider the number 2,458. How can you tell if a number is divisible by 2?
  If the number is even it is divisible by 2.
- How can you tell if the number is even?
  If it ends in 0, 2, 4, 6, or 8
- Why does this work? What about the rest of the number?

Ask students to write the number in expanded form considering place value.

$$2458 = 2000 + 400 + 50 + 8 = 2 \times 1000 + 4 \times 100 + 5 \times 10 + 8 \times 1$$

Consider each part of the number, the digit of 2 represents 2000, which equals 2×1000, this number is a multiple of 10 or power of 10.

$$2 \times 1000 = 2 \times 10^3.$$

All powers of 10 are even so 2000 is even. This is true for the 400 and 50 as well; they are also powers of 10. That means that $2000 + 400 + 50 = 2450$ is an even number because all of its addends are even. Thus $2450 + 8$ is even because 2450 is even and 8 is even. This is argument holds true for any number regardless of the number of digits.

- What about $3,578,647$?

$$3{,}578{,}647 = \underbrace{3{,}000{,}000 + 500{,}000 + 70{,}000 + 8000 + 600 + 40}_{} + 7$$

Each of these numbers is a power of 10 so they are all even. That means that the sum of all these number is also even.

This last digit is odd.

$$3{,}578{,}647 = 3{,}570{,}640 + 7$$

$$Even\ Number\ +\ Odd\ Number =\ Odd\ Number$$

2. Explain using words, pictures, examples, or equations why a number is divisible by 2 if the last digit is even and why a number is not divisible by 2 if the last digit is odd.
Every number that has more than one digit is a sum of a number that is a power of ten and a single digit. Every power of ten is an even number. Since an $even\ + even = even$ and $even + odd = odd$ you only have to consider whether the last digit is even or odd to determine if a number is divisible by 2.

3. Write 5 numbers, each with a different number of digits, which are divisible by 2.
Students may list any 5 numbers that are even, each with a different number of digits.

The same reasoning is used to determine if a number is divisible by 10 and 5.

Divisibility Rule for 10

- Consider the number $2{,}450$. How can you tell if a number is divisible by 10?
If it ends in zero
- Why does this work, what about the rest of the number?

Write the number in expanded form

$$2450 = 2000 + 400 + 50$$

All of the addends are divisible by 10.

Any number than ends in 0 is a power of 10 regardless of the number of digits.

- What about $3,578,647$?

$$3{,}578{,}647 = \underbrace{3{,}000{,}000 + 500{,}000 + 70{,}000 + 8000 + 600 + 40}_{} + 7$$

Each of these numbers are divisible by 10.

This last digit does not end in zero. It is not a factor of 10.

4. Explain using words, pictures, or equations why you know that a number is divisible by 10 if the last digit 0.
If the last digit is 0 that mean that the number is a power of 10 because if you write any number in expanded form each of its addends will be divisible by 10 and thus 10 is one of its factors.

5. Write 5 numbers, each with a different number of digits, which are divisible by 10.
Students may list any 5 numbers that end in 0, each with a different number of digits.

Divisibility Rule for 5

- Consider the number 2,455. How can you tell if a number is divisible by 5?
  If it ends in 0 or 5
- Why does this work, what about the rest of the number?

Write the number in expanded form

$$2455 = 2000 + 400 + 50 + 5$$

The first three addends are powers or multiples of ten and we know that powers of 10 are divisible by 5. The last digit is 5 so it is divisible by 5. Thus if the last digit is 5 it is divisible by 5 and if the last digit is 0 then the number is a power of 10 and it is also divisible by 5.

Any number than ends in 0 or 5 is a power of 5 regardless of the number of digits.

- What about $3,578,647$?

$$3{,}578{,}647 = \underbrace{3{,}000{,}000 + 500{,}000 + 70{,}000 + 8000 + 600 + 40}_{\text{Each of these numbers are divisible by 5 because they are powers of 10.}} + 7$$

This last digit does not end in 0 or 5, it is not divisible by 5

6. Explain in your own words why you know that a number is divisible by 5 if the last digit 0 or 5.
   If the last digit is 0 that means that the number is a power of 10 which is divisible by 5. If the last digit is 5 that means that the number is the sum of a number that is a power of 10 and 5. This makes it divisible by 5.

7. Write 5 numbers, each with a different number of digits, which are divisible by 5
   Students my list any 5 numbers that end in 5 or 0, each with a different number of digits.

Divisibility Rule for 4

- Consider the number 2,358. How can you tell if a number is divisible by 4?
  This one is not as intuitive; but a similar argument can be used using place value and powers of 100.
- Write the number in expanded form

$$2358 = 2\underbrace{000 + 300}_{} + 50 + 8$$

Divisible by 4 because they are multiples of 100 which is divisible by 4

Consider each addend; $2000 = 2 \times 1000$ this is divisible by 4 because we know that 1000 is divisible by 4. $300 = 3 \times 100$, this is divisible by 4 because we know that 100 is divisible by 4. In fact, any number that is a multiple of 100 is divisible by 4. Another way to look at the expression is $2358 = 2300 + 50 + 8 = 23 \times 100 + 50 + 8$

Since the first two addends are divisible by 4 then the number is divisible by 4 if the last two addends $(50 + 8)$ form a number that is also divisible by 4. We know that 58 is not divisible by 4. So 2,358 is not divisible by 4. Any number whose last two digits form a number that is divisible by 4 will be divisible by 4 regardless of the number of digits in the entire number.

- What about $3,578,644$?

$$3{,}578{,}644 = \underbrace{3{,}000{,}000 + 500{,}000 + 70{,}000 + 8000 + 600}_{\text{Each of these numbers are divisible by 4 because they are multiples of 100 which is divisible by 4.}} + \underbrace{40 + 4}_{\text{Consider the last 2 digits. 44 is divisible by 4}}$$

This number is divisible by 4 because the last two digits form a number that is divisible by 4.

8. Explain in your own words how you know that a number is divisible by 4 if the last two digits form a number that is divisible by 4.
   If the last two digits are divisible by 4 then the number is divisible by 4 because any digits before the last two represent numbers that are multiples of 100 and all multiples of 100 are divisible by 4.

9. Write 5 numbers, each with a different number of digits, which are divisible by 4.
   Sample answers are given: 16, 216, 2216, 22016, 220016. Students may list any 5 numbers where the last two digits form a number that is divisible by 4. Each number must have a different number of digits.

Divisibility Rule for 3 and 9

- Consider the number 1116. How can you tell if a number is divisible by 3 or 9?
- Write the number in expanded form

$$1116 = 1000 + 100 + 10 + 6$$

We have a sum of several powers of 10 and 6. However 3 and 9 do not divide any power of 10 or 100 or 1000, etc. Thus we cannot use the same reasoning as we have for our previous divisibility rules. But, do note that every power of ten can be expressed as a sum of a number that is divisible by 3 and 9 plus 1.

$$10 = 9 + 1 \quad 100 = 99 + 1 \quad 1000 = 999 + 1 \quad 10{,}000 = 9999 + 1 \quad etc$$

All of the addends that contain only digits of 9 are divisible by 9 and 3.

If we rewrite our number using addends with digits of 9 we get $1116 = (999 + 1) + (99 + 1) + (9 + 1) + 6$. We know that all of the addends that contain only digits of 9 are divisible by 3 and 9 so we only have to look at the addends that are leftover to see if they are divisible by 3 or 9. The leftovers are 1, 1, 1, and 6. Note that these leftover are the actual digits in the number. If we sum these leftovers we get $1 + 1 + 1 + 6 = 9$. Since this sum is divisible by 9 then the entire number is divisible by 9, likewise if this sum is divisible by 3 then the entire number is divisible by 3.

Any number whose digits add to a number that is divisible by 9 is divisible by 9 and any number whose digits sum to a number that is divisible by 3 is divisible by 3 regardless of the number of digits.

- What about 3567?

Consider the sum of the digits. $3 + 5 + 6 + 7 = 21$. 21 is divisible by 3 so this number is divisible by 3. However, 21 is not divisible by 9 so this number is not divisible by 9.

10. Write 3 numbers, each with a different number of digits, which are divisible by 3.
    Sample answers are given: 111, 6222, 93483 Students may list any 3 numbers whose digits sum to a number that is divisible by 3. Each number must have a different number of digits.

11. Write 3 numbers, each with a different number of digits, which are divisible by 9.
    Sample answers are given: 117, 6822, 93483 Students may list any 3 numbers whose digits sum to a number that is divisible by 9. Each number must have a different number of digits.

Divisibility Rule for 6

- Consider the number 2358. How can you tell if it is divisible by 6?
  Since 2 and 3 are both prime factors of 6 if a number is divisible by 2 and 3 it is divisible by 6.
  Use divisibility tests for 2 and 3, if both are satisfied then the number is divisible by 6.
- Is 2358 divisible by 2?
  Yes, it is even
- Is 2358 divisible by 3?
  Yes the sum of its digits 2+3+5+8=18 is divisible by 3.
  Thus 2358 is divisible by 6. Any number is divisible by 6 if it is divisible by 2 and 3 regardless of its number of digits.
- What about 39,441?
  It is not divisible by 2 because it is odd, it is divisible by three because the sum of its digits is 21.
  However it must by divisible by 2 and 3 in order for it to be divisible by 6, so it is not divisible by 6.

12. Write 3 numbers, each with a different number of digits, which are divisible by 6.
    Sample answers are given: 12, 24, 360
    Students may list any 3 numbers that are divisible by 2 and 3. Each number must have a different number of digits.

13. Summarize the Divisibility Rules below by completing each statement.

A number is divisible by:

- 2 if the last digit is even.
- 3 if the sum of the digits is divisible by 3.
- 4 if the last two digits form a number that is divisible by 4.
- 5 if the last digit is 0 or 5.
- 6 if it is divisible by both 2 and 3.
- 9 if the sum of the digits is divisible by 9.
- 10 if the last digit is 0.

15. Determine if each given number is divisibly by 2, 3, 4, 5, 6, 9, or 10. Justify your answer.
    a. 5040
       2-the number is even; 3-the sum of the digits is divisible by 3; 4-the last two digits form a number that is divisible by 4; 5-the number ends in 0; 6-the number is divisible by 2 and 3; 9-the sum of the digits is divisible by 9; 10-the number ends in 0.
    b. 955
       5-the number ends in 5

16. Circle all the numbers that are factors of 15,033,444.

(2) (3) (4) 5 (6) 9 10