0.1b Class Activity: Dividing Multi-Digit Numbers

Part 1: Roxy has received orders from several different schools requesting candy from her store for their school fundraiser. She has recorded the amount of each type of candy she has in stock and the number of schools that want each type of candy in the table below. She would like to give each school that requests a certain type of candy the exact same amount of candy to be fair. Study the table and without calculating, estimate if the amount of candy that each school receives is correct. Justify your answer.

Type of Candy	Truffle Troll Treats	Rainbow Drops	Lemon Swirly Pops
Amount in Stock	6255	154	4950
Number of schools requesting this candy	15	9	20
Amount of candy each school receives	417	14	247.5

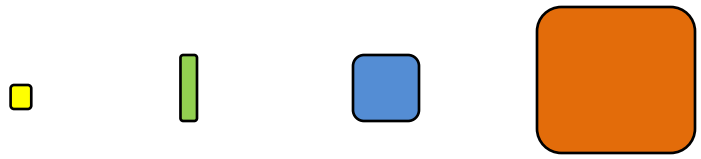
The amount of candy each school receives is correct for the Truffle Troll Treats and the Lemon Swirly Pops. You can determine this by using estimation. Ask students to share their estimation strategies with the class. Possible arguments might be that for the Truffle Troll Treats we know that 15 will go into 60 four times, this means that 15 will go into 6255 a little more than four hundred times. Thus 417 is a reasonable answer. For the Lemon Swirly Pops students might argue that 4950 is pretty close to 5000 and 20 goes into 5000 two hundred and fifty times. Since 247.5 is a little less than 250 this amount makes sense. We know that the amount received per school for the Rainbow drops it not correct because 9 will go into 100 almost exactly 11 times. 9 goes into 54 exactly 6 times. Thus the amount of candy that each school receives should be very close to $11 + 6 = 17$.

 Discuss with students how estimation plays an important role in division and also how good problem solving includes reflecting upon whether your answer makes sense given the context.

 The purpose of the next task is for students to solidify their understanding of how the division algorithm relates to place value using models. The models act as a mathematical tool to help students see this relationship. Assess their conceptual understanding of long division and how well they can relate the algorithm to place value. In other words, do they understand how and why the algorithm works? Once you are confident that students have this understanding help them to become fluent using the division algorithm. As student's work to do so they can make use of the mathematical structures that are used in long division.

Part 2:

Roxy has created a new cherry chocolate treat to sell in her store. She packages the cherry chocolates into tubes, boxes, and cases. Each tube contains 10 cherry chocolates, each box contains 10 tubes, and each case contains 10 boxes.



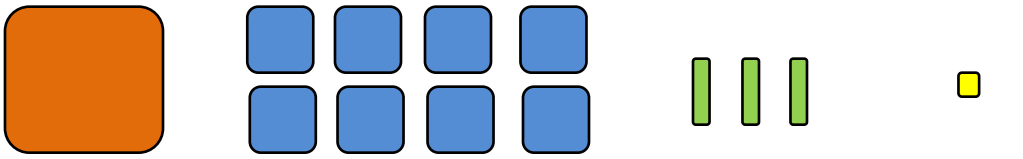
1 piece | Tube 10 pieces | Box 10 Tubes | Case 10 Boxes

**Figures not drawn to scale*

Roxy has made 1851 cherry chocolates and has received requests from 12 schools for the treats. She would like to give each school the same number of cherry chocolates. Determine how many cherry chocolates each school will receive and the number of cases, boxes, tubes, and pieces of cherry chocolates each school will receive.

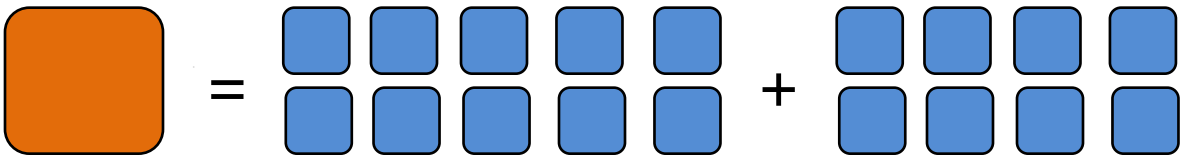
Ask students to estimate the solution first and share their estimation reasoning. The task is worked out in detail below showing how you can use place value models to solve the problem. In previous grades students used models to solve long division problems and have begun to relate these models to the long division algorithm. It is completely acceptable for students to solve with an area model or with partial quotients instead of with a place value model. As you work through this task with your students relate the models to the standard algorithm.

According to Roxy's packaging guidelines 1831 cherry chocolates would be packaged in 1 case, 8 boxes, 5 tubes, and 1 individual candy.



1 case | 8 boxes | 5 tubes | 1 piece

This is essentially breaking the dividend into its place value parts.
1 case cannot be divided amongst the 12 schools evenly. We must "unpack" the case. There are 10 boxes in a case.



1 case | 1 case is equivalent to 10 boxes | Add in the original 8 boxes.

This makes 18 boxes that can be distributed among the 12 schools.

If each school receives 1 box, which is really 100 pieces of candy, then 6 boxes are leftover.

Relate this to the algorithm.

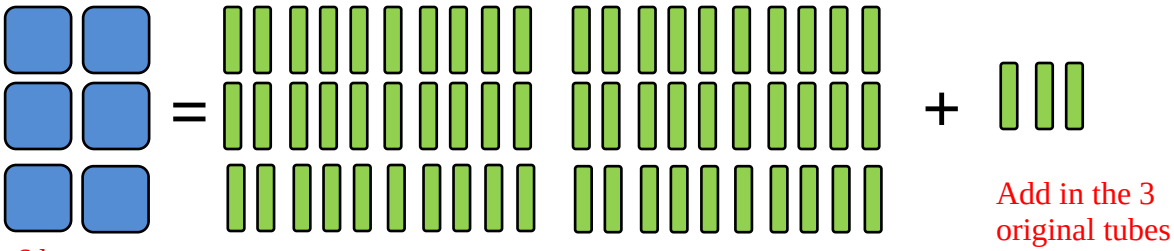
12 schools

1	2	3	4	5	6	7	8	9	10	11	12

Divisor | Quotient | Dividend

$$12\overline{)1831}\quad 1$$
$$-12$$
$$6$$

We must now divide the 6 boxes leftover amongst the 12 schools. Since the 6 boxes cannot be evenly distributed amongst the 12 schools than we must "unpack" the boxes into tubes. There are 10 tubes in a box.



6 boxes

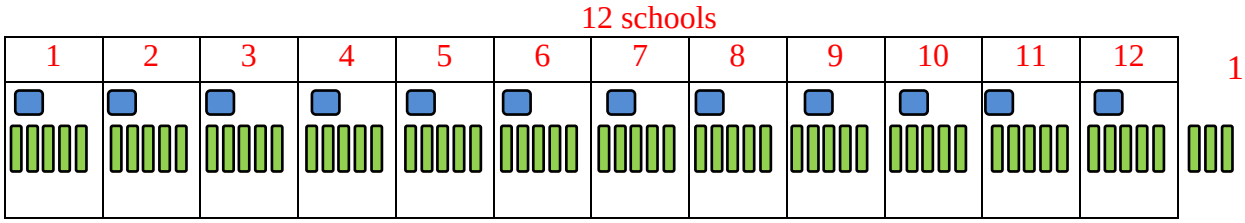
6 boxes is equivalent to 60 tubes

Add in the 3 original tubes

This makes 63 tubes that can be distributed among the 12 schools.

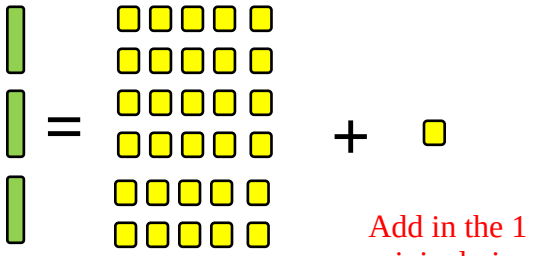
If each school receives 5 tubes, which is really 50 pieces of candy, then 3 tubes are leftover.

Relate this to the algorithm



$$\begin{array}{r} 15 \\ 12\overline{)1831} \\ -12\downarrow \\ \hline 63 \\ -\ 60 \\ \hline 3 \end{array}$$

We must now divide the 3 tubes leftover amongst the 12 schools. Since the 3 tubes cannot be "evenly" distributed amongst the 12 schools than we must "unpack" the tubes into pieces. There are 10 pieces in a tube.



3 tubes

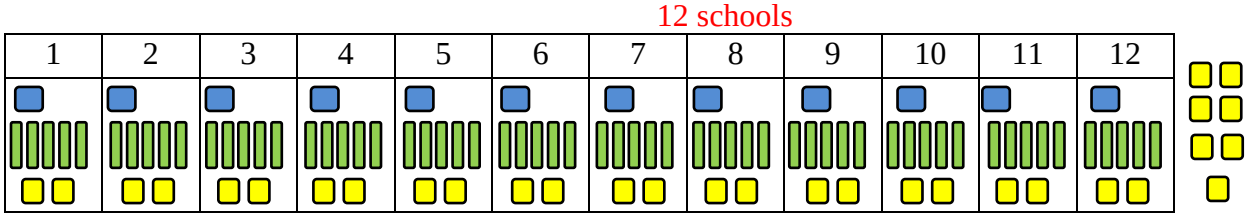
3 tubes are equivalent to 30 pieces

Add in the 1 original piece

This makes 31 pieces can be distributed among the 12 schools.

If each school receives 2 pieces than 7 pieces are leftover.

Relate this to the algorithm



$$\begin{array}{r} 152 \\ 12\overline{)1831} \\ -12 \\ \hline 63\downarrow \\ -\ 60 \\ \hline 31 \\ -\ 24 \\ \hline 7 \end{array}$$

This models shows that each school will receive 1 box, 5 tubes, and 2 pieces of chocolate cherries, this is 152 pieces of candy. Be sure to relate this to the algorithm, talking about how the 1 in the quotient is really 100 and the 5 is really 50. Roxy will have 7 pieces of candy leftover. When a division problem results in a quotient that is not a whole number students must be able to decide whether the context dictates them to report the quotient as a whole number with a remainder, as a decimal, or as a fraction. For this context it is not likely that Roxy would break the loose candies into small pieces to divide amongst the 12 schools. So reporting the quotient with a remainder is appropriate. However, students will practice writing quotients with decimals or fractions rather than remainders on the next page.

Part 3

After delivering all of the candy to the schools Roxy has earned $73 in tips for making the deliveries. She decides to split the tip money between she and her 3 employees. How much tip money will each person receive?

$$\begin{array}{r} 18.25 \\ 4\overline{)73.00} \\ -4\downarrow \\ \hline 33 \\ -32 \\ \hline 10 \\ -\ 8 \\ \hline 20 \\ -20 \\ \hline 0 \end{array}$$

For this context it makes sense to report the quotient as a decimal and not as a remainder because we want to know exactly how much money everyone will receive. If needed, draw a model to illustrate unbundling the remaining 1 dollar into 10 dimes, relate it to bringing down a zero in the standard algorithm. You can then illustrate unbundling the remaining 2 dimes into 20 pennies; this is shown in the algorithm by subtracting and bringing down the second zero. As students practice using long division paying attention to the repetition of "subtract" and "bring down" will aide in achieving fluency and ease when working with the algorithm.

You can also talk about writing the remainder as a fraction and how this fraction is related to the decimal. In this problem you have a remainder of 1. This remainder must be split amongst 4 people; you can represent this with the quotient $1 \div 4 = \frac{1}{4} = 0.25$

Each person will receive a tip of $18.25.

After making the calculation above Roxy realizes that she forgot to include the two delivery truck drivers when making her tip calculation. How much money will each person receive if she tips herself, her three employees, and the two delivery truck drivers?

$$\begin{array}{r} 12.1666... \\ 6\overline{)73.0000} \\ -6\downarrow \\ \hline 13 \\ -12 \\ \hline 10 \\ -\ 6 \\ \hline 40 \\ -36 \\ \hline 40 \\ -36 \\ \hline 40 \end{array}$$

For this context is makes sense to report the quotient as a decimal and not as a remainder because we want to know exactly how much money everyone will receive. There are two important topics of discussion in this problem. One is that is appears that the decimals will continue on forever. We know this because they begin to repeat themselves. Repeated decimals will be address thoroughly in 7th grade. Secondly, in relation to the context of money we round our answer up to the nearest hundredth because we cannot break up a penny.

Ask students to check this problem on their calculator so you can talk about why the answer on the calculator has a 7 at the end.

Each person will receive a tip of $12.66

Directions: Estimate each quotient. Then use the standard algorithm to find the exact quotient.
It may be helpful for you to review some benchmark numbers that aide in estimation when multiplying or dividing. Some of these benchmark numbers are 5, 10, 15, 20, 25, 30, 50, 100.
Also encourage students to make a list of the divisor's multiples to reference for each problem. See pages 10-12 of the Mathematical Foundation for additional detailed examples of the long division algorithm.

1.	2.	3.	4.
$14\overline{)322}$ $-\ 28$ 42 $-\ 42$ 0 Possible estimation reasoning might be that 14 is close to 15. We know that 15 goes into 300 twenty times so our quotient will be a little more than 20.	$12\overline{)5484}$ $-\ 48$ 68 $-\ 60$ 84 $-\ 84$ 0 Discuss possible estimation reasoning.	$115\overline{)8625}$ $-\ 805$ 575 $-\ 575$ 0 Possible estimation reasoning might be to round the divisor to 100 and the dividend to 8000. 100 goes into 800 eighty times. The quotient should be close to 80.	$205\overline{)21115}$ $-\ 205$ 61 $-\ 0$ 615 $-\ 615$ 0 In this problem discuss the role that the 0 in the quotient plays. This is obtained because 205 will not go into 61, this means than we must unbundle the 61 tens into 610 ones and then add the remaining 5 ones to get the 615. That is why there is a 0 in the tens place in the quotient.

5.

$$\begin{array}{r} 18.05 \\ 20\overline{)361.00} \\ -\,20 \\ \hline 161 \\ -\,160 \\ \hline 10 \\ -\,\;0 \\ \hline 100 \\ -\,100 \\ \hline 0 \end{array}$$

Discuss possible estimation reasoning and the zero in the tenths place of the quotient.

6.

$$\begin{array}{r} 121.2195121951 \\ 41\overline{)4970.0000000000} \\ -\,41 \\ \hline 87 \\ -\,82 \\ \hline 50 \\ -\,41 \\ \hline 90 \\ -\,82 \\ \hline 80 \\ -\,41 \\ \hline 390 \\ -\,369 \\ \hline 210 \\ -\,205 \\ \hline 50 \\ -\,41 \\ \hline 90 \\ -\,82 \\ \hline 80 \\ -\,41 \\ \hline 390 \\ -\,369 \\ \hline 210 \\ -\,205 \\ \hline 50 \end{array}$$

Discuss the repeating decimals, in this problem the decimal does not begin to repeat right away but eventually a pattern emerges.

7.

$$\begin{array}{r} 19.57386363 \\ 352\overline{)6890.00000000} \\ -\,352 \\ \hline 3370 \\ -\,3168 \\ \hline 2020 \\ -\,1760 \\ \hline 2600 \\ -\,2464 \\ \hline 1360 \\ -\,1056 \\ \hline 3040 \\ -\,2816 \\ \hline 2240 \\ -\,2112 \\ \hline 1280 \\ -\,1056 \\ \hline 2240 \\ -\,2112 \\ \hline 1280 \end{array}$$

Possible estimation reasoning might be to round the dividend to 7000 and the divisor to 350. 350 goes into 7000 twenty times. The quotient should be close to 20.

8.

$$\begin{array}{r} 18.125 \\ 6\overline{)108.750} \\ -\,6 \\ \hline 48 \\ -\,48 \\ \hline 07 \\ -\;\;6 \\ \hline 15 \\ -\,12 \\ \hline 30 \\ -\,30 \\ \hline 0 \end{array}$$

This problem is a natural extension to the previous problems. If you feel your students are not ready for it yet it will be revisited in section 0.01e.

As you work through the problems above relate the algorithm to place value, this will help the students not only become more fluent in the algorithm but help them to understand why the algorithm works. You can also encourage students to check their answer by multiplying the quotient by the divisor. If this product equals the dividend they have done the problem correctly.

Find, Fix, and Justify

9. Owen has completed the following division problem and has made a mistake. Find the mistake and explain what Owen has done wrong. Then solve the division problem correctly.

$$\begin{array}{r} 561.\overline{22} \\ 45\overline{)25245.00} \\ -\,225 \\ \hline 275 \\ -\,270 \\ \hline 55 \\ -\;\;45 \\ \hline 100 \\ -\;\;90 \\ \hline 100 \\ -\;\;90 \\ \hline 100 \end{array}$$

Owen should have brought down the 4 and not the 5.

$$\begin{array}{r} 561 \\ 45\overline{)25245} \\ -\,225 \\ \hline 274 \\ -\,270 \\ \hline 45 \\ -\;\;45 \\ \hline 0 \end{array}$$

This problem illustrates a common mistake when doing long division. It is important to keep your numbers lined up. If students struggle give them a piece of graph paper to do their calculations on so they write each number in the appropriate place value column.